THE RELATION BETWEEN PROBABILITY OF SUCCESS AND COMPETENCY LEVEL

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Abstract *The purpose of this paper is to derive a mathematical relationship between the probability of success and competency level of any individual in an activity. A deductive approach has been employed here to derive the mathematical relation. The probability of success has been found to be an exponential function of the competency level. The work of this paper would be the threshold for further research in constructing a valid and reliable scale of measurement for competency level in every activity. The individuals, by means of this relation, could prepare themselves in better a way to participate in any event and the HR professionals could more effectively modulate training programmes. In addition to this, the employers, instead of using the elimination process, may also use the relation for selecting appropriate human resources for the posts of their organisations. This paper is the first in its category to derive the mathematical relationship between probability of success and competency level.*

Keywords: *Probability of Success, Performance, Activity, Competency Level, Human Resources, Training.*

The rapid growth of the world's population, coupled with the steady decrease of natural resources, is causing more difficulties in the way to success in different spheres of our life. Still the success is the ultimate goal in every activity of all the individuals, groups, teams, institutions, organisations etc. Though the aim in an activity is a relative term for different performers, success is the accomplishment of the aim. In other words, success in an activity is relative for different performers. Under a certain circumstance, a set of proper responses of one is essential for the success of oneself in every activity. Individual responses in an activity arise from personal characteristics, some of which are inherited and the rest are acquired through learning. There have always been some formal and informal training arrangements, for developing such personal characteristics, in the history of mankind since the beginning of family concept as well as the concept of society.

In early twentieth century, psychologists considered intelligence as a prime factor behind every success. The intelligence scales, such as *'Binet-Simon Scale'* (1905), *'Stanford-Binet Intelligence Scale'* (1916), *'Wechsler Adult Intelligence Scale'* (1955) etc, were developed to measure the intelligence of individuals. The concept of intelligence had anyway dominated the human resource (HR) professionals until 'competency' was proved to be a better differentiator than intelligence for success or failure in all activities (McClelland, 1973). A generic definition of competency was given by a group of human resources scholars in Johannesburg, South Africa in 1995: "A competency is a cluster of related knowledge, skill, attitude (K, S, A) that affects a major part of one's jobs (a role or responsibility), that correlates with performance on the job, that can be measured against well-accepted standards, and that can be improved via training and development" (Parry, 1996). But, in the literature, the scholars have defined the competency in many different ways. The approaches for these definitions of competency are based on either: (i) 'observable performances' (Boam, and Sparrow, 1992; Bowden and Masters, 1993); (ii) 'the standard or quality of outcome of the person's performance' (Rutherford, 1995; Hager et al, 1994); or (iii) 'the underlying attributes of a person' (Boyatzis, 1982; Sternberg and Kolligian, 1990). The first and second categories of the approaches are related to the outputs or performance while the third category refers to the inputs or the underlying attributes (Hoffman, 1999). However the HR professionals, in present days, are using the concept of competency to identify the training needs; to prepare the training designs; to execute the designs of training and to evaluate the training modules for ensuring success of the human resources of an organisation in their respective duties or responsibilities.

In spite of all the differences in defining competency, it is well accepted that performance in an activity is correlated with the level of competency of the performers; but, theoretically, success or failure of a performer in an activity cannot be certain as the probability of human errors and other uncertainty factors in the universe also affect the success or failure. The probability of human errors is a consequence of the bio-psychological factors of the performer under a given circumstance. On the other hand, this probability of success in an activity cannot be the same for all performers with

different competency levels. Thus the probability of success in any activity should be related with the competency levels. But no effort has been found, in the literature, to establish any mathematical relationship between the probability of success and competency or competency level. An effort has been made in this paper to derive a mathematical relation between the probability of success and competency level of any performer in an activity.

1. METHODOLOGY

1.1. Probability of Success

One's success in an activity depends on the responses of oneself and the impacts of the rest of the universe. The individual responses arise from the set of personal characteristics which is labelled as competency. On the other hand, the impacts of the rest of the universe include impacts of everything except the responses of the performer. It is evident that if the impacts of the rest of the universe remain constant, the level of competency will be the only variable determinant of success or failure.

If both the responses of the performer and the impacts of the rest of the universe were certain, success or failure of an individual would also have been certain. But theoretically, the probability of human errors and the uncertainty in the impacts of the rest of the universe cannot be eliminated from any activity and, therefore the success or failure of an individual in every activity is a probabilistic matter. Under a certain explicit circumstance, this probability of success or failure in an activity cannot be the same for all performers of different competency levels; rather it varies with the competency levels of the performers. If the probability of success in an activity be the same for all the performers, the competency level of all the performers will be either equal or zero. Again, according to the fundamentals of the probability theory:

i. Both the probability of success (P_s) and the probability of failure (P_f) in any activity is less than one but greater than zero and
ii. The sum of the probability of success (P_s) and the probability of failure (P_f) in any activity is one. Therefore,

$$P_s + P_f = 1 \qquad (1)$$

1.2. Competency Level

The performances of different individuals, in any activity, may be different. The concept of different levels of performance (Dreyfus and Dreyfus, 1980) paved the way for the development of the concept of competency level. The graded scale of competency level is useful in categorising and comparing different performers in any activity according to their competencies. The competency levels, expressed in tune with Dreyfus and Dreyfus model, are discrete in nature. One cannot differentiate any individual, by the use of the concept of discrete competency level, from a group of performers with same competency level but different in competencies. Let us take an example, if two students A and B get Grade-A (80% - 100%) in a mathematics test, on the basis of this no one can distinguish one student as comparatively better performer than the other in the mathematics test. But if the marks of A and B respectively be expressed as 81% and 99% then one can easily say that B is comparatively better performer than A in the mathematics test. Thus the ratio or percentage based approach for competency level will be a more accurate one. In this context, *the competency level of a performer in an activity may be expressed as the ratio of the competency (C) of the performer in the activity to the highest possible competency (C_0) by any performer in the same.* Therefore the competency level (ψ) in an activity is,

$$\psi = \frac{C}{C_0} \qquad (2)$$

Here, the values of may vary from zero to one; i.e. $0 \le \psi \le 1$.

1.3. Relation Between Probability of Success and Competency Level

If a graph of the probability of success (P_s), under a certain explicit circumstance, in an activity be plotted against competency levels (ψ), the graph will be confined within the region $0 < P_s < 1$ and $0 \le \psi \le 1$. Let PQ be an arbitrary segment of the graph (Figure 1). The tangent at O (ψ, P_s), an arbitrary point on PQ, intersects the lines $P_s = 0$ and $P_s = 1$ at the points D and C respectively. AB is the perpendicular on the lines $P_s = 0$ and $P_s = 1$ from O. Then OB = the probability of success at the point O = P_s and OA= the probability of failure at the point O = $P_f = 1 - P_s$. Let us represent BD and AC with β_s and β_f respectively. Then the gradient of the tangent CD is,

$$\frac{\partial P_S}{\partial \psi} = \tan\angle CDB = \frac{OB}{BD} = \frac{P_S}{\beta_S} \qquad (3)$$

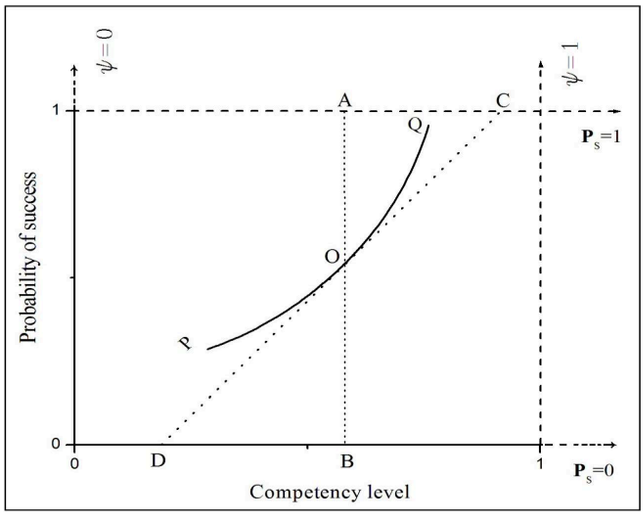
Again, from Δ AOC & Δ DOB,

$$\beta_s \times P_f = \beta_f \times P_s = \frac{1}{\tau} \qquad (4)$$

If the value of P_s and P_f be changed at any other point of the graph, the corresponding value of β_s and β_f will also change in accordance with the equation (4) as there has to be a specific relation between probability of success (P_s) and competency levels (ψ). Therefore will be constant in the activity. Now, from the equation (3) and (4), we get,

$$\frac{\partial P_S}{\partial \psi} = \tau P_s P_f = \tau P_S(1 - P_S) \qquad (5)$$

Figure 1: Representing an Arbitrary Graph of Probability of Success (P_S) Versus Competency Level (ψ)



The equation (5) is the differential equation of probability of success with respect to competency level. On integrating the equation (5) and rearranging the outcome, we get,

$$P_s = \frac{1}{(1+\mu.e^{-\tau\psi})} \quad (6)$$

The equation (6) is analogous to the formula of *logistic function* and it represents the mathematical relation between probability of success and competency level where the probability of success (P_s)is an exponential function of competency level (.) and is a constant.

Calculation of Constants: The constants of the equation (6), μ and τ, can be from the boundary values of ψ. If at ψ = 0, P_s be P_L and at ψ =1, P_S be P_H. Then the equation (6), at ψ = 0, reduces to,

$$\mu = \frac{(1-P_L)}{P_L} \quad (7)$$

Again, the equation (6) at ψ = 1, reduces to $\tau = \ln \frac{\mu P_H}{1-P_H}$ and again, by using the value of μ from the equation (7), τ can be expressed as,

$$t = In\left[\frac{(1-P_L)P_H}{(1-P_H)P_L}\right] \quad (8)$$

2. Illustration with an Example

Let us consider, for simplicity, an objective type test in which twenty questions are required to cover the entire topic. An uncertainty factor in the examination is introduced in form of multiple choice type objective questions having two options for each question. Now if the situation be so controlled that the probability of human error is limited around 0.01 (i.e. a participant may commit one mistake in answering 100 questions), the probability of selecting correct answer of each known question is 0.99. Now let us examine the following three cases:

1. The probability success (P_S) of scoring 100% marks: Here, one has to answer correctly all the twenty questions for scoring 100%. The probability of selecting correct answer of one unknown question = $\frac{1}{2}$. The lowest probability of success (P_L) is the probability of selecting correct answers of all the twenty questions while all of the questions are unknown. Therefore $P_L \left(\frac{1}{2}\right)^{20} \approx 9.54 \times 10^{-7}$. Again the highest probability of success (P_H) is the probability of selecting correct answers of all the twenty questions while all of the twenty questions are known. Therefore $P_H = (0.99)^{20} \approx 0.818$. Then, from the equations (7) and (8), we get $\mu \approx 1.05 \times 10^6$ and $\tau \approx 15.37$. Hence the equation (6) can be written as,

$$P_S = \frac{1}{\left\{1+\left(1.05\times10^6\right).e^{-(15.37)\psi}\right\}} \quad (9)$$

Now the values of probability of success (P_S) for different values of competency level can be calculated from the equation (9). For example, at ψ = 0.7, P_S = 0.0428. The values of P_S for different values of competency level (ψ) are shown in the Table 1.

2. The probability success ($P_{S'}$) of scoring at least 90% marks: Here, one has to answer correctly at least eighteen questions out of twenty questions to score at least 90% marks. The lowest probability of success (P_L) is the probability of selecting at least eighteen correct answer out of the twenty questions while all of the twenty questions are unknown. Therefore, according to the binomial formula of probability, $P_L = \sum_{18}^{20} C_{18}^{20} \times 0.5^{18} \times (0.5)^2 \approx 2.01 \times 10^{-4}$. Again the highest probability of success (P_H) is the probability of answering correctly at least eighteen questions out of the twenty questions while all of the twenty questions are known. Therefore $P_H = \sum_{18}^{20} C_{18}^{20} \times (0.99)^{18} \times (0.01)^2 \approx 0.999$. Then, from the equations (7) and (8), we get $\mu \approx 4.97 \times 10^3$ and $\tau \approx 15.42$. Hence the equation (6) may be written as,

$$P_{s'} = \frac{1}{1+\ 4.97\times10^3\ .e^{-(15.42)\psi}} \quad (10)$$

Now, the values of probability of success ($P_{s'}$) for different values of competency level (ψ) can be calculated from the equation (9), e.g. at ψ = 0.7, $P_{s'}$ = 0.907. The values of $P_{s'}$ for different values of competency level (ψ) are shown in the Table 1.

3. Probability of success ($P_{S''}$) of scoring 100% marks while 10% questions are unknown to every participant: Here, one has to correctly answer all the twenty questions

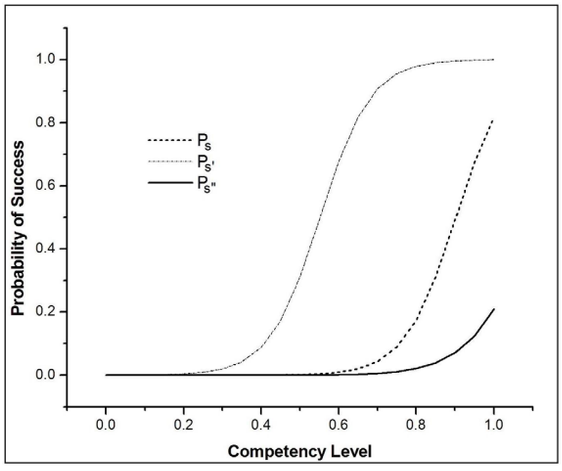
for scoring 100%. The lowest probability of success (P_L) is the probability of selecting correct answers of all the twenty questions while all of the questions are unknown. Therefore, $P_L = \left(\frac{1}{2}\right)^{20} = 9.54 \times 10^{-7}$ Again the highest probability of success (P_H) is the product of the probability of selecting all correct answers of the eighteen known questions and the probability of selecting correct answers of the both two unknown questions. Therefore $P_H = (0.99)^{18}\left(\frac{1}{2}\right)^{2} = 0.209$. Then, from the equations (7) and (8), we get $\mu \approx 1.05 \times 10^6$ and $\tau \approx 12.53$. Hence the equation (6) may be written as,

$$P_{S''} = \frac{1}{\left\{1+\left(1.05\times10^{6}\right).e^{-(12.53)\psi}\right\}} \qquad (11)$$

Now, the values of probability of success ($P_{s''}$) for different values of competency level (ψ) can be calculated from the equation (10), e.g. at $\psi = 0.7$, $P_{S''} = 0.00612$. The values of $P_{S''}$ for different values of competency level (ψ) are shown in the Table 1.

The data in Table 1 can be graphically represented as shown in the Figure 2 where the three graphs represent variation of the probability of success, P_s, $P_{s'}$ & $P_{s''}$, with competency levels.

Figure 2: Showing the Variation Of $P_{S'}$ $P_{S'}$ & $P_{S''}$ with Respect to Ψ



3. Discussion

The probability of success of any individual in an activity for a given competency can be calculated from the equation (6), if the lowest and the highest probability of success in that activity be determined. The lowest and the highest probability of success can be determined either from the results of large number of observations or can be calculated from the given conditions. In the example, mentioned above, the lowest probability of success (P_L) for the cases of P_s, $P_{s'}$ & $P_{s''}$ are respectively 9.54×10^{-7}, 2.01×10^{-4} & 9.54×10^{-7} and highest probability of success (P_H) for the same cases are respectively 0.818, 0.999 & 0.209 (Table 1). As all the values of P_L are very small and close to zero, the corresponding graphs of the probability of success, *viz* P_s, $P_{s'}$ & $P_{s''}$, as shown in the Figure 2, exponentially grow with and each of these will have at least one point of inflection; but the graph of $P_{S'}$, for which P_H is almost equal to one, will have two and finally becomes saturated.

Thus the nature of the graph of probability of success (P_S) against competency level may be different for different sets of boundary values of the equation (6). The variation of the probability of success graph may be as follows:

i. If both the lowest probability of success (P_L) and the highest probability of success (P_H) be approximately equal to zero and one respectively, the graph of the probability of success (P_S) will have two point of inflection and will be similar to the S-curve.

ii. If the lowest probability of success (P_L) be approximately equal to zero but the highest probability of success (P_H) be significantly less than one, the graph of the probability of success (P_S) will be exponential and will have a point of inflection to become active.

iii. If the lowest probability of success (P_L) be significantly greater than zero but the highest probability of success (P_H) be approximately equal to one, the graph of the probability of success (P_S) will be exponential and will have a point of inflection to become saturated.

iv. If both the lowest probability of success(P_L) and the highest probability of success (P_H) be significantly less than one, the graph of the probability of success (P_S) will tend to be a straight line between the points of the lowest probability of success (P_L) and the highest probability of success (P_H) with a gradient not equal to zero.

Further, in an activity, if the competency level be zero for all performers, the corresponding probability of success will

Table 1: Showing the Values of,P_S, $P_{S'}$ and $P_{S''}$ with Respect ψ

ψ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
P_S	9.54×10^{-7}	4.43×10^{-6}	2.06×10^{-5}	9.58×10^{-5}	4.45×10^{-4}	2.07×10^{-3}	9.53×10^{-3}	4.28×10^{-2}	0.172	0.492	0.818
$P_{S'}$	2.01×10^{-4}	9.39×10^{-4}	4.37×10^{-3}	2.01×10^{-2}	8.75×10^{-2}	0.309	0.677	0.907	0.979	0.995	0.999
$P_{S''}$	9.54×10^{-7}	3.34×10^{-6}	1.17×10^{-5}	4.09×10^{-5}	1.43×10^{-4}	5.02×10^{-4}	1.76×10^{-3}	6.12×10^{-3}	2.11×10^{-2}	7.02×10^{-2}	0.209

be a constant. The probability of success, for example, in flipping a fair coin for head is half.

4. Conclusion

A mathematical relation between probability of success and competency level has been established here and it is found that the probability of success is an exponential function of competency level. The human resource professionals, present days, are paying their concentration on competency based training (CBT) programmes but the standard and well accepted scales for measuring competency in different activities are still awaiting; in absence of such scales, the applications of the concept of competency in any field of activity will remain superficial. The work would be the threshold for further research in constructing a valid and reliable scale of measurement for competency as well as competency level in every activity. Thereby the relation between probability of success and competency level is expected to have a wide range of applications in both individual and organisational level. In individual level, there would be scopes of using the relation in educational, vocational and professional fields in near future. On the other hand, in organisational level, by means of this relation, the human resource professionals could more effectively modulate the need based training programmes and the employers or selectors, instead of using elimination process, could adopt competency based selection process to appoint or select the appropriate candidates for the posts of their organisations.

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