Advances in Engineering Science
Sect. C (2), April-June 2008, PP 7-10

A TURBO EQUALIZER WITH KALMAN FILTER BASED CHANNEL ESTIMATOR

Aruna Tripathy[1], Sant Saran Pathak[1] and Saswat Chakrabarti[2]

1 - Department of Electronics and Electrical Communication Engineering,
2 - GS Sanyal School of Telecommunications,
Indian Institute of Technology, Kharagpur, India-721 302

ABSTRACT

Turbo equalization (TEQ) is a base band signal processing technique that attempts reliable detection of data in a coded data transmission system subject to intersymbol interference (ISI) and additive white Gaussian noise (AWGN). The application of the turbo principle to equalization and decoding is called TEQ. The basic transmission system is a typical serial concatenated system (SCS). An SCS consists of two forward error correcting (FEC) encoders connected by means of a suitable interleaver. The outer FEC produces coded bits in response to the input data bits. The coded bits are interleaved so as to make them statistically independent. The interleaver is an essential component in a generic turbo receiver. The interleaved data feed a second stage called the inner encoder. The ISI channel serves as the inner encoder in the present work. It is viewed as applying redundancy on the interleaved data bits in the form of a linear convolution. The corresponding receiver consists of an equalizer and decoder connected by the deinterleaver. Both the equalizer and the decoder are configured as soft-in soft-out (SISO) signal processors. The equalizer takes as input the matched filter outputs and another information called extrinsic information provided by the decoder. The output of the equalizer is soft in nature as it is a ratio of two probabilities when binary phase shift keying is applied as the modulation technique. The equalizer is a trellis matched to that of the ISI channel. This is possible when we consider an ISI channel a finite state machine (FSM). The soft outputs are generated by the equalizer in terms of the log-likelihood ratio (LLR) on all the coded bits. These soft outputs serve as a priori to the FEC decoder, after suitable deinterleaving. The soft data estimates are computed by performing an ensemble average on the decoder soft outputs. The flow of extrinsic information between the equalizer and the decoder through interleaver and deinterleaver constitutes one iteration. These iterations are carried out for a predetermined number or convergence. The trellis based equalizer needs knowledge of the channel taps in order to compute the branch metrics and the transition probability. The literature on TEQ report the performance of the turbo equalizers for perfect channel estimates. However, in a practical scenario, the receiver needs to estimate the channel taps using some algorithm. We use a Kalman filter (KF) in a decision directed (DD) mode to estimate the channel and use these estimates subsequently to feed the trellis based equalizer. This receiver operates in a DD configuration due to the fact that, the data estimates formed at the decoder output serve as the training bits for the KF. When the TEQ is converging, the decoder produces more reliable data estimates that tend to approach their true values. The improvement in the data estimates improves the channel estimates by reducing the variance and the Kalman loop gain. The training data is treated as a stochastic signal that consists of a deterministic component and the random component. As the TEQ progresses with higher iterations, the random component is also reduced and this results in improved bit error rate (BER) at the decoder output.

Keywords: *Turbo, LLR, KF, DD, Soft Data Estimate*

I. INTRODUCTION

The next generation cellular networks are supposed to provide multimedia wireless services at data rates of the order of 10~100 Mbps. One of the challenges at the physical layer is the mitigation of ISI for single carrier systems in addition to AWGN. Channels with longer delay spreads exhibit ISI that spans multiple symbol intervals when the symbol duration is smaller than the coherence time of the channel. Turbo equalization [1] is a promising technique that takes care of significant ISI and AWGN in low signal to noise ratio (SNR) conditions and may be a suitable candidate for the non line-of-sight applications in a single carrier scenario. The excellent performance of a turbo equalizer reported in literature [1-4] for assume perfect channel estimates. However, in practice, the channel needs to be estimated and these estimated channel taps should be used for equalizing data. In this paper, we present a Kalman filter (KF) based channel estimator that operates in a decision directed mode along with a soft output Viterbi algorithm (SOVA)[5] based equalizer and decoder in order to realize a low complexity turbo equalizer.

II. Problem Formulation

The work reported in [6] also uses a KF based channel estimator receiver for a frequency selective static channel. However, [6] uses a BCJR [7] equalizer and a Log-MAP decoder. However, the use of the BCJR equalizer and a Log-MAP decoder requires high computational effort, significant latency and large storage of all the floating point probabilities as compared to other SISO algorithms [8,p 153, Table 5.1]. The literature on turbo equalizers focus mainly on the reduction of complexity for the equalizer and the decoder is neglected. For most of the works, the decoder is implemented as MAP and Log-MAP algorithms. However, the complexity of the overall receiver is determined by the complexity of the equalizer as well as the decoder. It has been noted in [9] that, a SOVA based TEQ may be more suitable from implementation point of view with an acceptable performance. Hence,we choose to work with a SOVA based TEQ. In a TEQ, the decoder has to make estimates of all coded data bits and hence for a rate $\frac{1}{n_0}$ code, N_0 decoders working in parallel actually make up the decoding unit. The use of N_0 MAP decoders increases complexity considerably. The choice of a SOVA TEQ results in a reduced latency, small storage and low computational complexity receiver as compared to MAP or Log-MAP BCJR based TEQs. The use of SOVA decoder reduces complexity (as compared to the MAP or Log-MAP) proportional to $N \times n_0 \times 2^K$ where N is the received data block size and K is the constraint length of the convolutional code. As the SOVA does not have to store the forward and backward probabilities, the storage of the floating point probabilities is reduced by $2 \times N \times n_0 \times 2^K$.

III. The Proposed Receiver

The basic transmitter is a kind of serial concatenated system that consists of an outer FEC code, an interleaver and the ISI channel represented as a discrete time transversal filter (DTTF) [10]. The ISI channel is viewed as a convolutional code of rate 1 with real valued outputs and is represented by L taps, h_i i = 0,...L–1. A zero mean AWGN with variance σ_w^2 is added to the transmitted signal at the baseband. A set of sufficient statistic for the purpose of equalization is obtained by sampling the matched filter output at baud-rate and is expressed as

$$z_k = \sum_{l=0}^{L-1} h_l b_{k-l} + w_k,\ k = 0,1,\ldots N-1 \qquad (1)$$

The transmitted data is b_k k = 0,1,...N – 1. The equalizer accepts these z_k s as input and an initial channel tap estimate and produces a vector of extrinsic information. The initial channel estimate is obtained by some training sequence. The soft output of the equalizer is computed as the minimum of the absolute additive path metric (APM) differences for the maximum likelihood (ML) path and its best competitor and it is called "log-likelihood ratio (LLR) in literature. A part of the LLR, called the extrinsic information is fed as *apriori* on all coded bits to the SOVA decoder after suitable deinterleaving. The decoder produces a vector of updated extrinsic information $\lambda_{Dec,n}$ for the n – th data bit and this is used, in turn, to generate the soft data bit estimates (SDBE). The entire vector of decoded soft symbols is used as the training vector for the Kalman channel estimator. The plant noise comes from the channel noise variance and is assumed to be known perfectly to the KF. The measurement noise is obtained by considering the transmitted signal a stochastic one, rather than deterministic and is expressed as

$$b_n = \overline{b}_n + \widetilde{b}_n \qquad (2)$$

where the known component is computed by considering the ensemble average of the SDBE and hence, is expressed as

$$\overline{b}_n = \tanh\left(\frac{\lambda_{Dec,n}}{2}\right) \qquad (3)$$

The argument in the RHS of (3) is the decoder extrinsic information corresponding to the nth bit. The variance of the random component as specified by $\widetilde{b}_n$ becomes the measurement noise. This is given as

$$v_n = 1 - \left(\bar{b}_n\right)^2 \tag{4}$$

As the turbo equalizer continues its iterations, this variance becomes very small and hence has been neglected from the third iteration onwards in our work from the second iteration.

Steps in the Kalmm*Gv(n)* (5)

$$z(n) = h^H(n)b(n) + w(n) = h^H(n)\left(\bar{b}(n) + \tilde{b}(n)\right) + w(n) = h^H(n)\bar{b}(n) + g(n) \tag{6}$$

where *h(n) = (h*$_{n,0,}$*................h*$_{n,L-1}$*),*

b(n) = (b(n),.....b(n – L + 1))^T and $\tilde{b}(n)$ is defined similarly.

$$g(n) = h^H(n)\tilde{b}(n) + w_0(n) \tag{7}$$

The autocorrelation of g(n) is found as

$$E[g(n+k)g^*(n)] = tr\{F^k R_h(n) K_{\tilde{b}}(n+k,n)\} + N_0/2\,\delta(k) \tag{8}$$

where tr(.) is the trace, and $R_h(n) = E[h(n)\,h^H(n)]$ (9)

and $K_{\tilde{b}}(n+k,n) = E\left[\tilde{b}(n+k)\tilde{b}^H(k)\right]$ (10)

The predictor-filter steps of the Kalman filter are as follows:

$$\hat{\mathbf{h}}(n|n-1) = \mathbf{F}\hat{\mathbf{h}}(n|n) \tag{11}$$

$$e(n|n-1) = z(n) - \hat{\mathbf{h}}(n|n-1)\bar{\mathbf{b}}(n) \tag{12}$$

$$\mathbf{k}(n) = \frac{\mathbf{P}(n|n-1)\mathbf{b}(n)}{tr\{Q_g(n)\} + \bar{\mathbf{b}}^H(n)\mathbf{P}(n|n-1)\bar{\mathbf{b}}(n)} \tag{13}$$

Filter Step

$$\hat{\mathbf{h}}(n|n) = \hat{\mathbf{h}}(n|n-1) + \mathbf{k}(n)e(n|n-1)^* \tag{14}$$

Error covariance update

$$\mathbf{P}(n+1|n) = \mathbf{F}\left[\mathbf{I} - \mathbf{k}(n)\bar{\mathbf{b}}^H(n)\right]\mathbf{P}(n|n-1)\mathbf{F}^H + \mathbf{G}\mathbf{Q}_v\mathbf{G}^H \tag{15}$$

$$\mathbf{P}(n|n-1) = E\left(\varepsilon(n|n-1)\varepsilon^H(n|n-1)\right) \tag{16}$$

$$\varepsilon(n|n-1) = \mathbf{h}(n) - \hat{\mathbf{h}}(n|n-1) \tag{17}$$

The Kalman gain vector is denoted by **k** (n).

IV. Results and Discussion

The proposed algorithm is evaluated for two different channels. A highly frequency selective static channel has been considered in this part of the paper. The input to the KF comes from the soft data bit estimates (SDBE) provided by the decoder. This is modeled as a stochastic signal having a deterministic component and a random component as shown by (2). The former is obtained by considering the ensemble average of the SDBE and the random component is the error that results after subtracting this deterministic component from the SDBE. The channel is modeled as a first order auto-regressive random process for which the state transition matrix $\mathbf{F} = \lambda^{-\frac{1}{2}}\mathbf{I}_{L\times L}$ and $\lambda = 0.999$ has been used in the simulation study. As all the soft decoded symbols are used as training bits, the variance in channel estimation error tends to be zero with respect to iterations and the channel estimates closely approach their true values.

The TEQ produces improved estimates of extrinsic information with respect to iterations in the waterfall region and hence the SDBE improves. Asymptotically, it approaches and as these are used as training symbols for the KF, the channel estimate improves. An illustrative performance of this turbo equalizer has been evaluated through simulation experiments. We consider a rate ½,recursive systematic convolutional (RSC) code, a random interleaver, a 3-tap static channel [0.407 0.815 0.407] as the transmitter. The SOVA decoder uses a traceback depth of 30 bits.

We note from Fig.1 that, the proposed receiver is able to converge to the true channel taps by the 5th iteration for this difficult-to-equalize channel. This is due to the improved data estimates and reduced variance associated with these estimates. The improvement in the BER is due to the improved data estimates and reduced variance associated with these estimates.

We find that, the performance improves with each iteration. But the rate of improvement becomes very small after four iterations for channel signal-to-noise ratios in the range of 1 to 4 dB. The channel estimates approach their true values at the end of the fourth iteration. This may be attributed to the relatively well behaved spectral nature of this channel as compared to the channel considered in Fig.1.

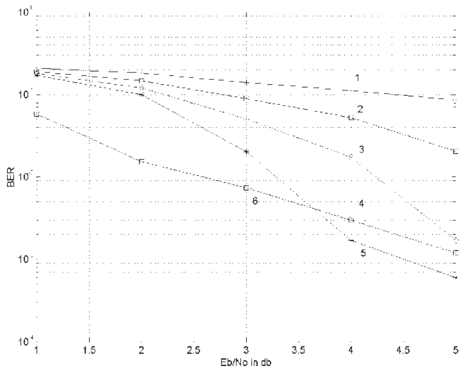

Figure 1 : Performance of SOVA based TED with Kalman filter based channel estimation, 1: Performance of SOVA TED with initial channel estimate [0 1 0], 2: Performance at second iteration with improved channel estimation, 3,4,5: Performance at 3rd, 4th, 5th iterations, 6: Performance of SOVA TED with perfect channel estimate at fourth iteration

We next consider a time varying channel which is represented by the Stanford University Interim (SUI) channel. This is also a three tap channel having an exponentially decaying profile. For each simulation run, a new channel is generated and this is estimated for a given block of data. We have considered 100 simulation runs.

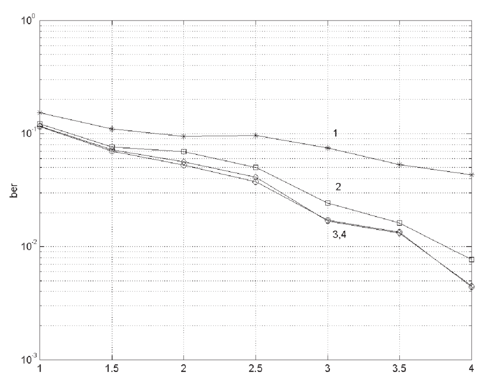

Figure 2 : Performance of the KF based TEQ in SUI channel for four iterations

For Fig.2, the proposed TEQ converges by the fourth iteration. This is due to the fact that this channel is a relatively good channel. The channels with bad instantaneous profile dominate the BER.

V. CONCLUSION

The KF channel estimator based TEQ working in the decision directed mode is observed to approach the performance as obtained for perfect channel tap knowledge at the end of the fifth iteration for a static ISI channel. Four iterations are required for the TEQ convergence in case of a moderate ISI channel.

REFERENCES

1. C. Laot, A.Glavieux and J.Labat, "Turbo Equalization: Adaptive Equalization and Channel Decoding Jointly Optimized", IEEE Journal on Selected Areas in Communications, vol.19, No.9 Sep.2001, PP 1744-1752
2. M.Tuchler, R.Koetter and A.C. Singer, "Turbo Equalization: Principles and New Results", IEEE Trans. on Comm.,vol.50,No.5, May 2002 PP 754-766
3. M.Tuchler, A.C.singer and R.Koetter, "Minimum Mean Squared Error Equalization Using A Priori Information", IEEE Trans. on Signal Processing,vol.50,No.3, March 2002, PP 673-683
4. Ralf Koetter, Andrew C. Singer and Michael Tuchler, " Turbo Equalization: An iterative equalization and decoding technique for coded data transmission", IEEE Signal Processing Magazine, PP 67-80, Jan. 2004
5. J.Hagenauer and P. Hoeher," A Viterbi algorithm with soft decision outputs and its applications," in Proc., IEEE Globecom Conf. Dallas, TX,Nov.1989 ,PP 1680-1686
6. S.Song, A.C.Singer and K.M. Sung, "Turbo Equalization with an Unknown Channel", VTC02, vol3,PP 2805-2808
7. 7: L.R. Bahl, J.Cocke, F.Jellinek and J.Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," IEEE Trans. on Info. Theory, Vol.20, No.2, Mar.1974, PP 284-287
8. B. Vucetic and J.Yuan, " Turbo Codes", Kluwer Academic Publishers, 2nd Printing, 2001
9. A. Tripathy, S.S. Pathak and S.Chakrabarti, "SISO based Turbo Equalization for Wireless Channels corrupted with ISI", 2nd IEEE Int. Conf. on Computing: Theory and Applications (ICCTA 07), ISI, Kolkata, 5-7th Mar, 2007, PP 81-86, Published by IEEE CS
10. G.D.Forney Jr.,"Maximum-likelihood sequence estimation for digital sequences in the presence of intersymbol interference", IEEE trans. on Info.Theory, vol.18, May 1972, PP 363-378