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Effects of Cash Conversion Cycle on Cash Management – A Study on IT Sector

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Abstract

This paper is made with an attempt to analyze the Cash Conversion Cycle and its effect on Cash Management of some well known IT companies (Philips, Asian Electronics Ltd., Wipro, CMC and Videocon) in India. The secondary data for analysis are retrieved from Capitaline database for ten years period from 2002-2011.Cash Conversion Cycle is an important concept. of Working Capital Management. The term CCC is used as a comprehensive measure of working capital because it considers the time gap between expenditure for the purchases of raw-materials and collection from sale of finished goods. So firm's short term assets and liabilities in a daily management play an important role in the success of the firm.

Keyword: Cash Conversion Cycle, Liquidity, Cash Management.

Introduction

The term Cash Conversion Cycle (CCC) can be considered a length of time between purchase of raw materials and collection of cash from debtors. In liquidity management Cash Conversion Cycle is an important parameter for measuring its efficiency. CCC of a company indicates the efficiency of managing working capital. Such measure can be used in benchmarking competitors or comparing companies. Cash Conversion Cycle is constructed by deducting the payable deferral period from the addition of inventory conversion period and receivable collection period.

We know about liquidity management which deals with management of current assets and liabilities. The main objective of maintaining current assets is that it can meet the current liabilities timely. Many firms take the advantage of external financing due to the difficulty in paying its short-term debt. But it should be remembered that it is not easy to collect such external financing easily, particularly in case of small firms. Another important factor in case of external financing is the cost of such borrowing. Sometimes, it is too expensive and as a result signifies the poor bottom line. Therefore, efficient liquidity management of the company helps its long-term prosperity and healthy bottom lines and more specifically to make the company remain solvent.

Cash Conversion Cycle or CCC(Moss and Stine, 1993) is such a useful technique by which we can easily and quickly assess the liquidity of the firm. It invariably measures the time lag between cash payments for purchase of inventories and collection of receivables from customers. Traditionally, some static balance sheet values such as current ratio and quick ratio are useful indicator of liquidity (Moss and Stine, 1993). But in case of CCC, it is a dynamic measure of continuous liquidity management, which comprises both balance sheet and income statement data with time dimension (Jose et al., 1996).

Cash Conversion Cycle = Days of Sales pending + Days of Sales in Inventory- Day of payables pending.

In the above equation the three variables on which CCC dependents are:

Days of sales pending = Accounts Receivables / Sales / 365

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Days of sales in inventory = Inventories / Cost of goods sold / 365

Days of payables pending = Accounts payables / Cost of goods sold / 365.

Cash Conversion Cycle can be positive or negative. A positive Cash Conversion Cycle indicates that the number of days a company is borrowing is less than the period awaiting payment from a customer. On the other hand, negative CCC implies the number of days a company received cash from sales before it must pay its suppliers (Hutchison et al., 2007, p. 42). More impressive thing is that the goal of every company is to minimize its CCC, if possible negative. Because shorter the CCC, the more efficient the company in managing its cash flow.

Review of Literature

1. Pedro Ortin Angel and Diego Prior made a study on accounting turnover ratios and cash conversion cycle. The main objective of the study was to deduce the amount of days spent completing an operational process from turnover ratios. This study provided additional tools for financial statements analysis in order to get accurate result or working capital management.
2. M. Deloof undertook a study on working capital management. His study was based on cash conversion cycle (CCC). In his study he used different measures relating to the time lag between expenditure for the purchase of raw materials and collection of sales of finished goods. He argued that the longer the time lag, the larger the investment in working capital.
3. L. Soenen (1993) carried out a research work on cash conversion cycle and its impact on profitability .In this study he argued that the degree of external financing affects the cash conversion cycle and the length of it.
4. M. Petersen and R. Rajan (1994) conducted a study on credit market competition on lending relationships. In this study they showed that companies that were rationed on credit by the banks were more likely to take trade credit loans.
5. M.J. Peel, N. Wilson and C. Howorth (2000) made a study on credit management in small firm sectors. In this study they suggested that small firms were associated with larger quantum of current assets as compared to large firms. They also argued that small firms had less liquidity, more volatile cash flows and largely relied on short-term debt.

Objectives of the Study

The presents study is prepared to make an in-depth analysis of the selected companies in IT sector in respect of their cash conversion cycle during the period of 2002-2011. Cash conversion cycle is one of the dynamic measures of liquidity of the organisation.

Generally, the main indicators of liquidity are current ratio (CR) and quick ratio(QR). Higher current ratio and quick ratio indicated good liquidity position of the organisation and vice-versa. High current ratio or quick ratio is the result of either high level of current assets (CA) or low level of current liabilities (CL). Cash conversion cycle (CCC) is another important indicator of liquidity. We developed the model of cash conversion cycle(CCC) on the basis of the model developed by Richards Laughlin (1980). According to him cash conversion cycle (CCC) is the sum of the receivables conversion period (RCP) and the inventory conversion period (ICP) minus the payment of deferral period (PDP). Therefore, smaller cash conversion cycle is expected because it helps the organisation to quickly recover its cash from the sale of its products. More cash in the hand of organisation means more liquid the organisation. Contrary, if the cash conversion cycle is high, it takes longer time to collect cash. Thus, a high cash conversion cycle is not desirable and it indicate liquidity problem. Hence, low cash conversion cycle is needed.

More specifically, the objectives of the study in this chapter are as follows:

1. To measure the cash conversion cycle of the selected companies from five different companies of IT sector with the help of receivables conversion period (RCP), inventory conversion period (ICP) and payment of deferral period (PDP).
2. To measure the average cash conversion cycle (CCC), deviation from the average of each of the selected companies using relevant statistical tools.
3. To rank the companies on the basis of average cash conversion cycle. Secondly, to rank the companies on the basis of consistency and finally to rank the companies on the basis of both average and consistency jointly.

4. To measure the degree of relationship between the cash conversion cycle and inventory turnover ratio (ITR), current ratio (CR), debtors turnover ratio (DTR), debtors more than six months and creditors turnover ratio (CTR) in each of the companies under study by using Pearson's simple correlation technique and to test such coefficients.
5. To analyse the joint impact of earning capability (RONW), size of the organisation and cumulative profitability (Shareholders' Fund) on the cash conversion cycle of the companies with the help of appropriate statistical measure (i.e. multiple regression analysis) and to test the significance of such regression coefficients.
6. Finally, to examine whether the finding of the study conform to the theoretical argument or not.

Methodology of the Study

Five popular companies from IT sector have been selected for this study. The data of the selected companies for the period 2002 to 2011 used in this study have been taken from the secondary sources i.e. Capitaline Corporate Database of Capital Market Publishers (I) Ltd. Mumbai. For the purpose of our study different companies from IT sector are selected following the purposive sampling procedure. Receivable conversion period, inventory conversion period and payment of deferral period are used to measure the cash conversion cycle. Shorter cash conversion cycle means better liquidity position of the organisation. Here, we established the relationship between CCC and debtors more than six months, CCC and CR, CCC and inventory turnover ratio, CCC and debtors turnover ratio, and CCC and creditors turnover ratio. Debtors more than six months mean debtors from whom money is collected after six months. It is riskier to the organisationas it blocks cash for long period and reduces the liquidity position. Liquidity of the organisation has been represented by the current ratio which is obtained by dividing the current assets to current liabilities. Efficiency of the inventory management has been measured by inventory turnover ratio (ITR) which is the ratio between cost of goods sold and average stock. Debtors' turnover ratio (DTR) is the ratio of credit sales to average receivables. Organisation's ability to avail credit facility from suppliers has been measured by creditors' turnover ratio (CTR) which is the ratio of credit purchase to average payables.

Profitability, size of the organisation and cumulative profitability can influence the cash conversion cycle of the organisation. In this study profitability has been measured by return on net worth (RONW); size of the organisation has been represented through the amount equal to the log value of total assets. Shareholders fund has been selected in this study as cumulative profitability which consists of equity share capital and reserve surpluses. The log value of shareholders' fund represents the cumulative profitability.

For analyzing the data statistical tools like arithmetic mean, standard deviation coefficient of variation etc. and statistical techniques like Pearson's simple correlation analysis and multiple regression analysis and statistical test like 't' test, Sandler 'A' test have been applied at appropriate places.

Findings of the Study

From Table 1 it is found that in IT sector the cash conversion cycle of Philips India Ltd. is highest in the year 2006(49.84 days) and lowest in the year 2003 (39.03 days). On an average it is 44.1 days. Philips India Ltd. followed a mixed trend during the study period. Therefore, we can say that the liquidity position of the company is sound in 2003 as compared to other years and worst in 2006.

In case of Asian Electronics Ltd. the CCC is highest in the year 2010 (483.04 days) and lowest in the year 2002 (94.31days). On an average it is 275 days. From the table it is clear that there has been a steady increase in CCC except in the year 2007 and 2011. Hence, the short term debt paying capacity of the organisation detoriated from year after year.

In case of Wipro the picture is totally different. It comprises very small days in CCC. The highest CCC has been identified in the years 2006 (57.97 days) and 2008 (44.93 days). The CCC of Wipro followed a mixed trend. It started with 54.85 days and ended with 56.27 days. On an average it is 51.3 days. Thus the company registered a moderate CCC during the study period. It maintained a moderate liquidity position during the study period.

CMC maintained moderate CCC during the study period. It started with 39.66 days and ends with 18.65 days. The highest CCC is found in the year 2005 (78.23 days) and smallest CCC is depicted in the year 2010 (16.36 days). On an average it is 43 days. So the liquidity position of the study period except the years 2005, 2003 and 2006.

From Table 1, it has been found that the CCC of the Videocon Ltd. is highest in the year 2006 (109.63 days) and lowest in the year 2009 (52.70 days). It also registered a mixed trend of CCC during the study period. On an average it is 86.8 days. The company maintained a moderate CCC during the study period.

Therefore, among five companies from IT sector the liquidity position of CMC is good as compared to others. It also portrays that the liquidity management system of CMC is sound, than other four companies in that sector.

In Table 2 the values of average cash conversion cycle (CCC) of the companies under study have been ascertained by applying arithmetic mean, and consistency of CCC has also been measured by using the coefficient of variation (CV) of their cash conversion cycle. The industry wise ranks have been assigned to the selected companies both in respect of average and consistency.

In the companies of IT sector selected in this study, the average CCC of CMC Ltd. is the lowest, followed by Philips, Wipro, Videocon and Asian Electronics Ltd respectively in that order. The table also reveals that in respect of consistency of designing CCC, Philips captured the top position and followed by Wipro, Videocon, Asian Electronics Ltd and CMC Ltd respectively. Considering both average and consistency aspects together, Philips occupied the first rank whereas Wipro got the second position, followed by CMC, Videocon and Asian Electronic Ltd.

Coefficient of Correlation is the measurement of degree of association between two variables. A positive value of 'r' indicated high values of one variable are generally associated with the high values of other variables and low values with low values. In Table 3 an effort has been made to measure the degree of relationship between Cash Conversion Cycle (CCC) and each of the factors related with CCC such as inventory turnover ratio (ITR), current ratio (CR), debtors turnover ratio (DTR), debtors more than six months (Debt > 6 Months) and creditors turnover ratio(CTR). To test the significance of such coefficient, 't' test has been applied.

According to Richards-Laughlin, CCC is the sum of receivables conversion period (RCP) plus the inventory conversion period (ICP) minus the payment deferral period (PDP) i.e. CCC = RCP + ICP – PDP

It is the indicator of liquidity. On the other hand, current ratio is also another indicator of liquidity. In this study we measure the closeness of CCC and CR. Generally, they are perfectly, positively correlated.

Inventory turnover ratio is calculated with the help of the following formula,

ITR = Cost of Goods Sold / Average Stock

This ratio indicates the efficiency of the inventory management of the organisation. Higher inventory turnover ratio indicates sound inventory management of the organisation. On the other hand low inventory turnover ratio implies excess inventory level which ultimately block up the cash. Therefore, higher inventory turnover ratio means lower cash conversion cycle, which is desirable. So inventory turnover ratio negatively related with CCC.

Debtors turnover ratio is the ratio of credit sales to average receivables i.e.

DTR = Credit sales / Average receivables

Higher debtors turnover ratio means shorter average collection period. It indicates the efficiency in collection of debt. On the other hand, lower debtors turnover ratio portrays longer average collection period. Higher the collection period means longer Cash Conversion Cycle. It is not expected. So, higher debtors turnover ratio indicates lower Cash Conversion Cycle. Hence, Debtors turnover ratio is negatively. related with CCC.

Debtors more than six months means payments are not received from debtors for more than six months. It is riskier to the orgnisation to collect funds from them. More time for getting the money from debtors signifies inefficient debt collection policy and it affects the Cash Conversion Cycle. Higher the period due from debtors larger is the Cash Conversion Cycle. Larger the Cash Conversion Cycle means in efficient Cash management system. CCC is positively related with debtors, which is pending for more than six months.

Creditors turnover ratio is the ratio of credit purchase to average payables, i.e.

CTR = Credit purchase / Average Payables

The creditors turnover ratio indicates number of times average creditors turnover in relation to purchase for the year. It reflects firm's ability to avail of credit facility from suppliers. A low creditors turnover ratio is apparently favourable as in that case firm enjoys lengthy credit period. It requires less working capital. Contrary, higher creditors turnover ratio indicates that the firm is to pay its suppliers immediately after purchase. For this, the firm has to invest more working capital. Low creditor turnover ratio means shorter Cash Conversion Cycle. Therefore, CCC is positively related with the creditors turnover ratios.

Table 3 shows that the correlation coefficient between CCC and ITR in Philips, Asian Electronics Ltd., Wipro, CMC Ltd. and Videocon are 0.387, 0.815, 0.2, 0.812 and (-) 0.011 respectively. Out of which the correlation coefficient between CCC and ITR in Philips, Asian Electronics Ltd., Wipro and CMC Ltd. are positive and correlation coefficient in case of Asian Electronics Ltd. and CMC Ltd. are statistically significant both at 5% and 1% level of significance. It implies that the strength of positive association between CCC and ITR in Asian Electronics Ltd. and CMC Ltd. is highly significant. Higher ITR helps to minimize CCC. But the correlation coefficient between CCC and ITR in Videocon is negative and statistically insignificant both at 5% and 1% level of significance.

Table 3 exhibits that in IT sector the correlation coefficient between CCC and CR in Philips, Asian Electronics Ltd and CMC Ltd. are 0.102, 0.972 and 0.761. Out of which the correlation coefficient between CCC and CR of Asian Electronics Ltd. and CMC Ltd. are statistically significant at 5% level. It indicates that the degree of positive association between CCC and CR in Asian Electronics Ltd. and CMC Ltd. selected for our study is highly significant. On the other hand the correlation coefficient between CCC and CR in Wipro and Videocon are (–) 0.052 and (-) 0.654 out of which the correlation coefficient between CCC and CR of Videocon is statistically significant at 5% level. It implies negative association between CCC and CR.

Table 3 depicts that in case of IT sector the correlation coefficient between CCC and DTR in Philips, Asian Electronic Ltd., Wipro and CMC Ltd. are 0.677, 0.982, 0.034 and 0.449 respectively out of which the correlation coefficient between CCC and DTR in Philips and Asian Electronics Ltd. are statistically significant at 5% level. It implies positive association between CCC and DTR in these four companies. On the other hand the correlation coefficient between CCC and DTR in Videocon is (–) 0.042 which is not at all significant. It implies negative relationship between CCC and DTR in Videocon.

In case of IT sector, the correlation coefficient between CCC and Debtors more than six months in Philips, Asian Electronics Ltd and Wipro are (–) 0.140, (–)0.659 and (–) 0.152 respectively out of which the same in case of Asian Electronics Ltd is statistically significant at 5% level. It implies the negative association between CCC and debtors more than six months in Philips, Asian Electronics Ltd. and Wipro. Debt collection policy which minimizes CCC of these three companies is good. On the other hand, the correlation coefficients between CCC and Debtors more than six month are positively low in CMC Ltd. and Videocon which are 0.271 and 0.314 respectively. It implies positive association between them which are not desirable.

Table 3 shows that in IT sector the correlation coefficient between CCC and CTR in Philips, Wipro, CMC Ltd and Videocon are (–) 0.266, (–) 0.519, (–)0.598 and (–) 0.276 respectively. It implies that the association between CCC and CTR is negative in Philips, Wipro, CMC Ltd. and Videocon Ltd. It is due to low creditors turnover ratio which is desirable to shorten the CCC. On the other hand the correlation coefficient between CCC and CTR in Asian Electronics Ltd. is positive which does not portrayed the theoretical conception of lower CTR to shorter CCC.

In Table 4 an attempt has been made to assess the influence profitability, size of the organization and cumulative profitability on Cash Conversion Cycle. In this study, return on net-worth (RONW) has been taken as the measure of owners' profitability, log value of total assets has been taken as the measure of size of the organisation, and shareholder's fund has been taken as the measure of cumulative profitability. The linear regression equation has been fitted in this study is CCC = b0 + b1 RONW + b2Size of Org. + b3 Shareholders' fund, where b0 is the value of intercept term (constant) and b1, b2 and b3 are the slopes of the line i.e. the regression coefficient of CCC on RONW, size of org. and Shareholders' fund. This regression equation has been tested by't' test.

Table 4 shows that in case of IT sector for one unit increase in RONW the CCC of Philips stepped up by 5.84 units, which is found to be statistically significant at 5% level. The table also shows that for one unit increase in the size of the organisation the CCC of Philips go up by 3.817 units which is found to be statistically insignificant at 5% level. It is found from Table 4 that for one unit increase in shareholders fund the CCC is increased by only 0.578 units which is also statistically insignificant at 5% level. It implies that the influence of RONW, size of organisation and shareholders' fund on CCC is positive. The coefficient of determination (R2) makes it clear that 65.5 % of the variation of the company's CCC is accounted for by the variation in RONW, size of Orginasation and shareholders' fund.

It has been found from Table 4 that for one unit increase in RONW the CCC of Asian Electronics Ltd. increased by only 0.004 unit which is not statistically significant. Table 4 shows that for one unit increase in size of the organisation the CCC of Asian Electronics Ltd. goes down by 8.636 units which is also statistically insignificant at 10% level. Similarly, Table 4 reveals that for one unit increase in shareholders' fund the CCC of Asian Electronic Ltd. stepped up by 6.503 units which is not at all significant at 5% level. The influence of RONW and shareholders' fund on CCC is positive but not significant whereas size of the organisation negatively influenced the CCC of the company. The coefficient of determination (R2) makes it clear that 54.0 % of the variation of the company's CCC is accounted for by the variation in RONW, Size of Org and Shareholders' fund.

It is found from Table 4 that for one unit increase of RONW the CCC of Wipro went up by 0.018 units which is not statistically significant at 5% level. The table also shows that for one unit increase in size of the organisation the CCC of Wipro stepped up by 8.036 units which is also not statistically significant at 5% level. Table 4 shows that for one unit increase of shareholders' fund the CCC of Wipro rapidly went down by 8.401 units and it is also not statistically significant at 5% level. It implies that the influence of RONW and size of the organisation on CCC of the company is positive but insignificant whereas the influence of shareholders fund on CCC is negative in the company. The coefficient of determination (R2) makes it clear that 28.5 % of the variation of the company's CCC is accounted for by the variation in RONW, size of organisation and shareholders' fund.

Table 4 shows that for one unit increase in RONW the CCC of CMC Ltd. stepped up by 0.219 units which is statistically insignificant at 5% level. Table 4 also reveals that for one unit increase in size of the organisation, the CCC of CMC Ltd. went down by 21.359 units which is also statistically insignificant at 5% level. Table 4 shows that for one unit increase in cumulative profitability the CCC of the company rapidly increased by 35.883 units which is also statistically not significant at 5% level. It indicates the influence of RONW and cumulative profitability on CCC of the company is positive but insignificant while the influence of size of the organisation is negative on the CCC of the company. The coefficient of determination (R2) makes it clear that 77.8 % of the variation of the company's CCC is accounted for by the variation in RONW, size of organisation and shareholders' fund.

It has been revealed from Table and 4 that for one unit increase in RONW the CCC of Videocon Ltd. is stepped down by 0.437 units which is statistically insignificant at 10% level. On the other hand, Table 4 also reveals that for one unit increase in size of the organisation the CCC of Videocon went down by 20.615 units which is not statistically significant at 5% level. Table 4 also shows that for one unit increase in size of the organisation the CCC of Videocon decreased by 24.704 units which is statistically insignificant at 5% level. It implies that the influence of size of the organisation on CCC of the company is positive whereas the impact of profitability as well as cumulative profitability is negative on the CCC of the company. The coefficient of determination (R2) makes it clear that 58.6 % of the variation of the company's CCC is accounted for by the variation in RONW, Size of org and Shareholders' fund.

Therefore, from table-4 it is found that only in case of Philips the influence of RONW, size of the organisation and cumulative profitability on CCC is positive.

In Table 5, an estimation of the values of CCC of Philips, in different years under study has been made by using the linear regression equation of CCC on RONW, size of the organisation and shareholders' fund. The deviations between the actual CCC and the estimated CCC have also been found out and these deviations have been tested by applying Joseph Sandler's 'A' test. Table 5 discloses that in 2003, 2004, 2009, 2010 and 2011 the actual CCC is in excess than the estimated CCC whereas in all the other years under study there was shortage in CCC. This table also depicts that the computed value of Sandler's 'A' is 6848209 which is not found to be statistically significant at 10% level. This insignificant variation between the actual and expected CCC reflects the efficiency in managing Cash conversion Cycle on the part of the company during the study period.

In Table 6, an estimation of the values of CCC of Asian Electronics Ltd., in different years under study has been made by using the linear regression equation of CCC on RONW, size of the Organisation and shareholders' fund. The deviations between the actual CCC and the estimated CCC have also been found out and these deviations have been tested by applying Joseph Sandler's 'A' test. Table 6 discloses that in 2002, 2007 and 2010 the actual CCC is in excess than the estimated CCC whereas in all the other years under study there was shortage in CCC. This table also depicts that the computed value of Sandler's 'A' is 47464.69 which is not found to be statistically significant at 10% level. This insignificant variation between the actual and expected CCC reflects the efficiency in managing Cash conversion Cycle on the part of the company during the study period.

In Table 7, an estimation of the values of CCC of Wipro, in different years under study has been made by using the linear regression equation of CCC on RONW, size of the organisation and shareholders' fund. The deviations between the actual CCC and the estimated CCC have also been found out and these deviations have been tested by applying Joseph Sandler's 'A' test. Table 7 discloses that in 2004,2005,2006,2009 and 2010 the actual CCC is in excess than the estimated CCC whereas in all the other years under study there was shortage in CCC. This table also depicts that the computed value of Sandler's 'A' is 2860.202 which is not found to be statistically significant at 10% level. This insignificant variation between the actual and expected CCC reflects the efficiency in managing Cash conversion Cycle on the part of the company during the study period.

In Table 8, an estimation of the values of CCC of CMC Ltd., in different years under study has been made by using the linear regression equation of CCC on RONW, size of the organisation and shareholders' fund. The deviations between the actual CCC and the estimated CCC have also been found out and these deviations have been tested by applying Joseph Sandler's 'A' test. Table 8 discloses that in 2002, 2004, 2005, 2006, 2010 and 2011 the actual CCC is in excess than the estimated CCC whereas in all the other years under study there was shortage in CCC. This table also depicts that the computed value of Sandler's 'A' is 6829.537 which is not found to be statistically significant at 10% level. This insignificant variation between the actual and expected CCC reflects the efficiency in managing Cash Conversion Cycle on the part of the company during the study period.

In Table 9, an estimation of the values of CCC of Videocon, in different years under study has been made by using the linear regression equation of CCC on RONW, size of the organisation and shareholders' fund. The deviations between the actual CCC and the estimated CCC have also been found out and these deviations have been tested by applying Joseph Sandler's 'A' test. Table 9 discloses that in 2003, 2008, 2009 and 2011 the actual CCC is in excess than the estimated CCC whereas in all the other years under study there was shortage in CCC. This table also depicts that the computed value of Sandler's 'A' is 32207.63 which is not found to be statistically significant at 10% level. This insignificant variation between the actual and expected CCC reflects the efficiency in managing Cash Conversion Cycle on the part of the company during the study period.

Conclusion

Liquidity management deals with the management of current assets and current liabilities. Its main objective is to maintain current assets in such a way that it can meet the current liabilities timely. Many firms take the advantage of external financing due to the difficulty in paying its short-term debt. But the firm cannot collect such external financing easily, particularly in case of small firms. External financing is costly. So, the efficient liquidity management of the company helps its long-term prosperity and healthy bottom lines and more specifically to make the company remain solvent.

Cash Conversion Cycle (CCC) is such a useful technique by which we can easily and quickly assess the liquidity of the firm. It invariably measures the time lag between cash payments for purchase of inventories and collection of receivables from customers. CCC is a dynamic measure of continuous liquidity management, which comprises both balance sheet and income statement data with time dimension.

An individual firm's CCC is helpful but from industries stand point it is crucial for a company to evaluate its performance regarding CCC and assess opportunities for improvement because the length of CCC may differ from industry to industry.

The liquidity position of Siemens Ltd. is the best as compared to other companies selected under this study. Considering both average and consistency aspects of CCC together, Hawkins and Khaitan together occupied the first place among other companies selected in this study.

From correlation point of view, we can say that most of the companies under study followed a positive relationship between CCC and ITR. On the other hand, in general the study failed to provide any clear indication in favour of the basic accepted principle that better current ratio, shorter the Cash Conversion Cycle. It is an accepted principle that higher the DTR, shorter the debtors collection period and also shorter the CCC. Thus, in the majority of companies under the study followed the accepted principle of higher the DTR, shorter the CCC. There is a generally accepted principle that higher the periods for collection of debt, higher the CCC and vice-versa. The study conform to the generally accepted principle of shorter debt collection period, shorter the CCC as most of the companies. Theoretically, CCC is negatively related with CTR as higher CTR means higher need working capital which is not desirable. So most of the companies in our study have conform that principle.

From the linear regression equation it is clear that the effect of RONW on CCC is positive. But the impact of size of the organisation on CCC is negative in most of the cases. In most of the companies the impact of shareholders' fund (cumulative profitability) on CCC are positive.

For measuring the efficiency in managing the cash conversion cycle we made the Sandler 'A' test and it signifies that all the companies selected under this study, except Siemens maintained efficient cash management system in relation to Cash Conversion Cycle.

Table 1: Analysis of Cash Conversion Cycle of Selected Companies of IT sector

COMPANIES	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	AVG
PHILIPS	45.94	43.52	40.86	39.69	49.26	49.84	43.17	42.22	39.03	47.36	44.1
ASIAN EC. LTD.	387.45	483.04	474.89	237.52	234.79	240.73	236.37	198.12	166.31	94.31	275
WIPRO	56.27	49.13	47.12	44.93	45.42	57.97	54.44	51.96	51.19	54.85	51.3
CMC	18.65	16.36	36.46	26.79	41.62	59.72	78.23	43.78	68.31	39.66	43
VIDEOCON	82.47	81.75	52.70	90.92	103.78	109.63	97.12	92.99	76.47	80.17	86.8
Source: Compiled and computed from 'Capitaline Corporate Database' of Capital Market Publishers (I) Ltd., Mumbai.											

Table 2: Ranking on the basis of Average and Consistency of Cash Conversion Cycle Of the Selected Companies from IT Sector

COMPANIES	AVG	SD	RANK OF AVG	COEFF. OF VAR.	RANK OF COEFF.	TOTAL RANK	OVER ALL RANK
PHILIPS	44.1	3.8582	2	8.7509	1	3	1
ASIAN EC. LTD.	275	129.932	5	47.188	4	9	5
WIPRO	51.3	4.58772	3	8.938	2	5	2
CMC	43	20.5023	1	47.726	5	6	3
VIDEOCON	86.8	16.0867	4	18.533	3	7	4
Source: Compiled and computed from 'Capitaline Corporate Database' of Capital Market Publishers (I) Ltd., Mumbai.							

Table 3: Karl Pearson's Simple Correlation Analysis between CCC and ITR, CR, DTR, Debt > 6 Months and CTR of the Selected Companies from IT Sector

COMPANY	*CCC & ITR*		*CCC & CR*		*CCC & DTR*		*CCC & DEBT > 6 MONTHS*		*CCC & CTR*	
	(r)	't' Value	(r)	't' Value	(r)	't' Value	(r)	't' Value	(r)	't' Value
PHLIPS	0.387	1.187	0.102	0.29	0.677*	2.602	–0.140	–0.4	–0.266	–0.78
ASIAN EC. LTD.	0.815**	3.978	0.972**	11.7	0.982**	14.71	–0.659*	–2.48	0.548	1.853
WIPRO	0.2	0.577	–0.052	–0.15	0.034	0.096	–0.152	–0.43	–0.519	–1.717
CMC	0.812**	3.935	0.761*	3.318	0.449	1.421	0.271	0.796	–0.598	–2.11
VIDEOCON	–0.011	–0.03	–0.654*	–2.45	–0.042	–0.12	0.314	0.935	–0.276	–0.812

Note: Figures in the parentheses indicate 't' values.
* Correlation is significant at the 5% level (2tailed).
**Correlation is significant at the 1% level (2tailed).

Source: Compiled and computed from 'Capitaline Corporate Database' of Capital Market Publishers (I) Ltd., Mumbai.

Table 4: Analysis of Multiple Regression of CCC on RONW, Size of Org. and shareholders' Fund of the Selected companies of IT Sector.

Regression Equation is CCC = b_0 +b_1 RONW + b_2 Size of Org. + b_3 Shareholders' Fund

COMPANY	*PARTIAL REGRESSION COEFFICIENT*			*CONSTANT*	*R2ED*
	RONW	*SIZE OF THE ORGANISATION*	*SHAREHOLDERS' FUND*		
PHLIPS	5.840 (3.343)**	3.817 (0.467)	0.578 (0.086)	–11.328 (–1.542)	0.655
ASIAN	0.004 (0.162)	–8.636 (–2.289)**	6.503 (1.628)	8.344 (2.387)	0.540
WIPRO	0.018 (0.349)	8.036 (1.190)	–8.401 (–1.104)	7.590 (1.763)	0.285
CMC	0.219 (1.248)	–21.359 (–0.510)	35.883 (1.094)	–28.226 (–0.903)	0.778
VIDEOCON	–0.437 (–2.190)*	20.615 (1.661)	–24.704 (–1.753)	15.762 (2.302)	0.586

Note: Figures in the parentheses indicate 't' values.
* Correlation is significant at the 5% level (2tailed).
**Correlation is significant at the 1% level (2tailed).

Source: Compiled and computed from 'Capitaline Corporate Database' of Capital Market Publishers (I) Ltd., Mumbai.

IT SECTOR

Estimation of the Cash Conversion Cycle with the Help of the Linear Regression Equation CCC = b_0 + b_1 RONW + b_2 Size of Org. + b_3 Shareholders' Fund

Table 5: Estimation of CCC of PHILIPS

Year	*Actual CCC (in times)[A]*	*Exp CCC (in times)[E]*	*Excess/ Shortages of CCC [A-E]*
2002	7.706	7.771	-0.065074
2003	9.351	9.272	0.0783906
2004	8.645	8.135	0.5095313
2005	8.453	8.702	-0.2486
2006	7.323	7.748	-0.424621

Contd...

Year	*Actual CCC (in times)[A]*	*Exp CCC (in times)[E]*	*Excess/ Shortages of CCC [A-E]*
2007	7.409	8.266	-0.856677
2008	9.196	9.204	-0.008269
2009	8.932	8.486	0.4460695
2010	8.386	8.153	0.2337811
2011	7.944	7.609	0.3349833

Joseph Sandler's 'A' value between Actual and Expected CCC is 6848209(being insignificant at 10% level).

Table 6: Estimation of CCC of ASIAN ELEC. LTD.

Year	Actual CCC (in times)[A]	Exp CCC (in times)[E]	Excess/ Shortages of CCC [A-E]
2002	3.870089	2.506959	1.36313
2003	2.194609	2.216991	-0.0224
2004	1.842305	2.071204	-0.2289
2005	1.544166	2.300864	-0.7567
2006	1.516184	1.863895	-0.3477
2007	1.554581	0.984306	0.57027
2008	1.536709	1.579327	-0.0426
2009	0.768588	0.818832	-0.0502
2010	0.755619	0.575438	0.18018
2011	0.942052	1.598626	-0.6566
Joseph Sandler's 'A' value between Actual and Expected CCC is 47464.69(being insignificant at 10% level).			

Table 7: Estimation of CCC of WIPRO

Year	Actual CCC (in times)[A]	Exp CCC (in times)[E]	Excess/ Shortages of CCC [A-E]
2002	6.4862	7.158085	–0.67188
2003	7.4286	7.572554	–0.14392
2004	7.7455	7.715085	0.030442
2005	8.1227	7.627834	0.494889
2006	8.0349	6.879136	1.155813
2007	6.2961	6.870358	–0.57427
2008	6.7043	6.927956	–0.22368
2009	7.0242	6.876526	0.147646
2010	7.1295	6.875959	0.253585
2011	6.6539	7.091742	–0.43783
Joseph Sandler's 'A' value between Actual and Expected CCC is 2860.202(being insignificant at 10% level).			

Table 8: Estimation of CCC of CMC

Year	Actual CCC (in times)[A]	Exp CCC (in times)[E]	Excess/ Shortages of CCC [A-E]
2002	9.203169	6.790998	2.412171
2003	5.342943	6.392952	-1.05001
2004	8.335911	8.03794	0.29797
2005	4.665217	3.831013	0.834204
2006	6.111838	5.837851	0.273987
2007	8.768736	11.79088	-3.02214
2008	13.62248	14.19268	–0.5702
2009	10.01072	15.22408	–5.21336
2010	22.3038	17.27861	5.025196
2011	19.56068	18.4466	1.11408
Joseph Sandler's 'A' value between Actual and Expected CCC is 6829.537(being insignificant at 10% level).			

Table 9: Estimation of CCC of VIDEOCON

Year	Actual CCC (in times)[A]	Exp CCC (in times)[E]	Excess/ Shortages of CCC [A-E]
2002	4.552621	4.611233	–0.05861
2003	4.772946	3.778378	0.994569
2004	3.924955	4.650819	–0.72586
2005	3.757934	4.389756	–0.63182
2006	3.329231	3.475695	-0.14646
2007	3.516988	4.07422	–0.55723
2008	4.014285	3.583862	0.430423
2009	6.925456	6.027242	0.898213
2010	4.464735	5.103893	–0.63916
2011	4.425527	4.00042	0.425106
Joseph Sandler's 'A' value between Actual and Expected CCC is 32207.63(being insignificant at 10% level).			

Table value of 'A' statistic with (n-1), i.e. 9 degrees of freedom at 10% level is 0.368.

Source: Compiled and computed from 'Capitaline Corporate Database' of Capital Market Publishers (I) Ltd., Mumbai.

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APPENDIX A: NUMERICAL SOLUTION OF SYSTEM OF EQUATIONS AND THE USE OF MATRIX ALGEBRA

Introduction

In power system problems, such as load flow and transient stability, we encounter both linear and nonlinear systems of equations. Since the order of these equations is very high, it is important to have fast, efficient, and accurate numerical algorithms. In this appendix we outline a few of these techniques.

We are concerned with solving the following system of *n* simultaneous equations in the *n* unknowns $x_1, \ldots, x_n$.

$$\begin{aligned} f_1(x_1, x_2, \ldots, x_n) &= 0 \\ f_2(x_1, x_2, \ldots, x_n) &= 0 \\ \vdots \qquad & \quad \vdots \\ f_n(x_1, x_2, \ldots, x_n) &= 0 \end{aligned} \tag{A.1}$$

A special case of the above, when $f_1, \ldots, f_n$ are linear in $x_i's$, results in a system of linear equations of the form

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= u_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= u_2 \\ \vdots \qquad & \quad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= u_n \end{aligned} \tag{A.2}$$

More compactly we have

$$AX = U \tag{A.3}$$

where ***A*** is the matrix of coefficients a_{ij}

$$X = [x_1, \ldots, x_n]^T \;\; and \;\; U = [u_1, \ldots, u_n]^T$$

System of equations of type (A.1) that are nonlinear are invariably solved using *iterative methods*. System of linear equations of type (A.2) can be solved using either (1) the iterative technique, or (2) employing *direct methods* involving a finite number of operations. We first deal with iterative methods.

Iterative Methods for Linear System of Equations

Jacobi methods

Consider Equation (B.2). Solving for $x_i's$

$$\begin{aligned} &x_1 \quad (u_1 - a_{12}x_2 - a_{13}x_3 - \quad - a_{1n}x_n)/a_{11} \\ &x_2 \quad (u_2 - a_{21}x_1 - a_{23}x_3 - \quad - a_{2n}x_n)/a_{22} \\ &x_n \quad (u_n - a_{n1}x_1 - a_{n2}x_2 - \quad - a_{n,n-1}x_{n-1})/a_{nn} \end{aligned} \tag{A.4}$$

Equation (B.4) can be written compactly as

$$x_i = \frac{\left(u_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij}x_j \right)}{a_{ii}}, \quad i = 1, 2, \ldots, n \tag{A.5}$$

In the above formulation it is assumed that $a_{ii} \neq 0$. Usually the equations are reordered if necessary so that *diagonal dominance* holds, i.e., each diagonal element a_{ii} is larger in magnitude than the magnitudes of other entries in row *i* and column *i*.

For iterative solution, we assume initial values for $x_i's$ and substitute them in the right-hand side of Equations (A.5). The resulting new approximations for the $x_i's$ are resubstituted on the right-hand sides of Equation (A.5) and the process repeated. After several iterations if the process has converged the $x_i's$ computed will show very little change. A criterion for convergence can be set up as follows:

$$\operatorname{Max}_i \left| x_i^{(k+1)} - x_i^{(k)} \right| < \varepsilon \tag{A.6}$$

where the superscripts (*k*) and (*k* + 1) denote the *k*th and (*k* +1)th iteration, respectively.

Gauss-Seidel iterative method

In this method the linear system of equations are arranged in the form of Equation (A.5). In the iterations, the new values of $x_i's$ are always used on the right-hand sides as soon as they are computed. In contrast, in the Jacobi method, the new values are not used until iteration for all the n components is completed. Symbolically, if we denote the initial guess as $x_i^{(0)}\,(i=1,2,\ldots,n)$, then the values obtained at the end of the first iteration are denoted by $x_i^{(1)}\,(i=1,2,\ldots,n)$ and so on. The values at the end of the kth and $(k+1)$th iteration are denoted by $x_i^{(k)}$ and $x_i^{(k+1)}\,(i=1,2,\ldots,n)$. The iterates are defined by

$$x_i^{(k+1)} = \begin{pmatrix} u_{ii} - a_{i1}x_1^{(k+1)} \ldots - a_{i,i-1}x_{i-1}^{(k+1)} \\ -a_{i,i+1}x_{i+1}^{(k)} \ldots - a_{in}x_n^{(k)} \end{pmatrix} / a_{ii},\; i=1,2,\ldots,n \qquad \text{(A.7)}$$

Both the Jacobi and Gauss-Seidel methods of solving the system of equations iteratively are sometimes referred to as *relaxation* methods (the errors in the initial starting values of x_i's are decreased or relaxed as computation continues).

Direct Method of Solving Linear System of Equations

This method is known as the *Gaussian elimination method* and involves a finite number of operations in arriving at the solutions. Consider the system of three equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= u_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= u_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= u_3 \end{aligned} \qquad \text{(A.8)}$$

As a first step, divide the first equation by a_{11}. Next, replace the second equation by adding to it the first equation multiplied by $-a_{21}/a_{11}$. Similarly, replace the third equation by adding to it the first equation multiplied by $-a_{31}/a_{11}$. The result is the system

$$x_1 + \frac{a_{12}}{a_{11}}x_2 + \frac{a_{13}}{a_{11}}x_3 = \frac{u_1}{a_{11}} \qquad \text{(A.9)}$$

$$\begin{aligned} &(a_{22} - a_{12}a_{21}/a_{11})x_2 + (a_{23} - a_{13}a_{21}/a_{11})x_3 \\ &= u_2 - u_1a_{21}/a_{11} \\ &(a_{32} - a_{12}a_{31}/a_{11})x_2 + (a_{33} - a_{13}a_{31}/a_{11})x_3 \\ &= u_3 - u_1a_{31}/a_{11} \end{aligned}$$

More compactly, these equations can be written as

$$\begin{aligned} x_1 + b_{12}x_2 + b_{13}x_3 &= v_1 \\ a_{22}^{(1)}x_2 + a_{23}^{(1)}x_3 &= u_2^{(1)} \\ a_{32}^{(1)}x_2 + a_{33}^{(1)}x_3 &= u_3^{(1)} \end{aligned} \qquad \text{(A.10)}$$

Now divide the second equation in Equation (A.10) by $a_{22}^{(1)}$ and replace the third equation by adding to it the second equation multiplied by $-a_{32}^{(1)}/a_{22}^{(1)}$. The result is as follows:

$$\begin{aligned} x_1 + b_{12}x_2 + b_{13}x_3 &= v_1 \\ x_2 + \left(a_{23}^{(1)}/a_{22}^{(1)}\right)x_3 &= u_2^{(1)}/a_{22}^{(1)} \\ \left[a_{33}^{(1)} - \left(a_{23}^{(1)}a_{32}^{(1)}/a_{22}^{(1)}\right)\right]x_3 &= \left(u_3^{(1)} - u_2^{(1)}a_{32}^{(1)}/a_{22}^{(1)}\right) \end{aligned} \qquad \text{(A.11)}$$

More compactly the above sets of equations can be written as

$$\begin{aligned} x_1 + b_{12}x_2 + b_{13}x_3 &= v_1 \\ x_2 + b_{23}x_3 &= v_2 \\ a_{33}^{(2)}x_3 &= u_3^{(2)} \end{aligned} \qquad \text{(A.12)}$$

As a final trivial step divide the third equation by $a_{33}^{(2)}$ and in terms of the new coefficients the system of equations becomes

$$\begin{aligned} x_1 + b_{12}x_2 + b_{13}x_3 &= v_1 \\ x_2 + b_{23}x_3 &= v_2 \\ x_3 &= v_3 \end{aligned} \qquad \text{(A.13)}$$

where $v_3 = u_3^{(2)}/a_{33}^{(2)}$.

In matrix notation Equation (A.13) becomes

$$\begin{bmatrix} 1 & b_{12} & b_{13} \\ 0 & 1 & b_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \qquad \text{(A.14)}$$

The process of computation in arriving at the form (A.13) or (A.14) is termed as *forward substitution*. To find x_1, x_2, and x_3 we perform the *backward substitution* procedure starting from the third equation in (A.14) as follows:

$$\begin{aligned} x_3 &= v_3 \\ x_2 &= v_2 - b_{23}v_3 \\ x_1 &= v_1 - b_{12}x_2 - b_{13}x_3 \end{aligned} \qquad \text{(A.15)}$$

the preceding procedure can be generalized to a system of n equations as follows:

Define

$$b_{kj} = \frac{a_{kj}^{(k-1)}}{a_{kk}^{(k-1)}} \qquad j \geq k+1 \qquad \text{(A.16)}$$

$$v_k = u_k^{(k-1)} / a_{kk}^{(k-1)} \qquad \text{(A.17)}$$

$$a_{ij}^{(k)} = a_{ij}^{(k-1)} - a_{ik}^{(k-1)} b_{kj} \qquad i, j \geq k+1 \qquad \text{(A.18)}$$

$$u_i^{(k)} = u_i^{(k-1)} - a_{ik}^{(k-1)} v_k \qquad i \geq k+1 \qquad \text{(A.19)}$$

The final solution is:

$$x_i = v_i - \sum_{j=i+1}^{n} b_{ij} x_j, \qquad i = n, n-1, \ldots, 2, 1 \qquad \text{(A.20)}$$

System of Nonlinear Equations

The system of nonlinear equations of the type (A.1) are invariably solved iteratively using efficient computational schemes. There are basically two competitive algorithms: (1) the Jacobi and Gauss-Seidel method and (2) the Newton-Raphson method. The relative merits of both of these methods will be discussed at the end.

Jacobi and Gauss-Seidel method

If the n Equations (A.1) are expressible in the form

$$\begin{aligned} x_1 &= \Phi_1(x_1, x_2, \ldots, x_n) \\ x_2 &= \Phi_2(x_1, x_2, \ldots, x_n) \\ &\vdots \\ x_n &= \Phi_n(x_1, x_2, \ldots, x_n) \end{aligned} \qquad \text{(A.21)}$$

Then a straightforward iterative technique is possible.

More compactly, let

$$x_i = \Phi_i(x) \quad i = 1, 2, \ldots, n \qquad \text{(A.22)}$$

Let the initial values be assumed to be $x_i^{(0)}$. Then the successive estimates of the vector $x^{(k)} = \left[x_1^{(k)}, \ldots, x_n^{(k)}\right]^T$ can be obtained by (1) the Jacobi method, or (2) the Gauss-Seidel method.

In Jacobi's method, the iterates are defined by

$$x_i^{(k+1)} = \Phi_i\left(x_1^{(k)}, x_2^{(k)}, \cdots, x_n^{(k)}\right) \quad i = 1, 2, \ldots, n \qquad \text{(A.23)}$$

In the Gauss-Seidel method, the recently computed values are used in the equations, i.e.,

$$x_i^{(k+1)} = \Phi_i\left(x_1^{(k+1)}, x_2^{(k+1)}, \cdots, x_{i\ 1}^{(k+1)}, x_i^{(k)}, x_{i+1}^{(k)}, \ldots, x_n^{(k)}\right) \qquad \text{(A.24)}$$
$$i = 1, 2, \ldots, n$$

The iterations are continued until the maximum difference between consecutive values of $x_i\,(i = 1, 2, \ldots, n)$ is less than a predetermined value ε, i.e.,

$$\operatorname*{Max}_i \left| x_i^{(k+1)} - x_i^{(k)} \right| < \varepsilon \qquad \text{(A.25)}$$

Newton-Raphson method

The Newton-Raphson method is applied directly to Equation (A.1). Before explaining the method to system of nonlinear equations, it is helpful to illustrate for a nonlinear equation in one variable.

Consider the scalar nonlinear equation $f(x) = 0$ in the variable x. Let the initial guess be $x^{(0)}$. Expand $f^{(x)}$ around $x^{(0)}$ in a Taylor series. Then the equation becomes

$$\begin{aligned} & f\left(x^{(0)}\right) + \left(x - x^{(0)}\right) f'\left(x^{(0)}\right) \\ & + \frac{1}{2!}\left(x - x^{(0)}\right)^2 f''\left(x^{(0)}\right) + \ldots = 0 \end{aligned} \qquad \text{(A.26)}$$

Neglecting second-order terms and higher, we get

$$f\left(x^{(0)}\right) + \left(x \quad x^{(0)}\right) f'\left(x^{(0)}\right) = 0 \qquad \text{(A.27)}$$

Solving this equation for x, we get an improved estimate as

$$x^{(1)} = x^{(0)} - \frac{f'\left(x^{(0)}\right)}{f\left(x^{(0)}\right)} \qquad \text{(A.28)}$$

Generalizing for the kth iteration we get

$$x^{(k+1)} = x^{(k)} \quad \frac{f'\left(x^{(k)}\right)}{f\left(x^{(k)}\right)} \qquad \text{(A.29)}$$

Convergence can be tested as before or by an alternative criterion such as $|f| < \varepsilon$. Indeed if the iterations converge, then $f \to 0$. The equation $f(x) = 0$ may have several solutions, in which case the converged value is the one closest to the initial guess.

System of nonlinear equations — Consider first a system of two equations

$$\begin{aligned} f_1(x_1, x_2) &= 0 \\ f_2(x_1, x_2) &= 0 \end{aligned} \qquad \text{(A.30)}$$

Expansion in a Taylor series around $x_1^{(0)}$ and $x_2^{(0)}$, and the retention of only the linear terms yields

$$f_1\left(x_1^{(0)}, x_2^{(0)}\right) + \left(x_1 - x_1^{(0)}\right)\frac{\partial f_1}{\partial x_1}\bigg|_0 + \left(x_2 - x_2^{(0)}\right)\frac{\partial f_1}{\partial x_2}\bigg|_0 = 0$$

$$f_2\left(x_1^{(0)}, x_2^{(0)}\right) + \left(x_1 - x_1^{(0)}\right)\frac{\partial f_2}{\partial x_1}\bigg|_0 + \left(x_2 - x_2^{(0)}\right)\frac{\partial f_2}{\partial x_2}\bigg|_0 = 0 \quad \text{(A.31)}$$

Casting the above equations in matrix form and solving for x_1 and x_2, we obtain

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_0 \begin{bmatrix} \left(x_1 - x_1^{(0)}\right) \\ \left(x_2 - x_2^{(0)}\right) \end{bmatrix} = \begin{bmatrix} -f_1\left(x_1^{(0)}, x_2^{(0)}\right) \\ -f_2\left(x_1^{(0)}, x_2^{(0)}\right) \end{bmatrix} \quad \text{(A.32)}$$

Hence, the first approximation for x_1 and x_2 yields

$$\begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{bmatrix} = \begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \end{bmatrix} - \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_0^{-1} \begin{bmatrix} f_1\left(x_1^{(0)}, x_2^{(0)}\right) \\ f_2\left(x_1^{(0)}, x_2^{(0)}\right) \end{bmatrix} \quad \text{(A.33)}$$

Using the values of $x_1^{(1)}$ and $x_2^{(1)}$, improved estimates $x_1^{(2)}$ and $x_2^{(2)}$ can be obtained following a similar procedure. The iterations are continued until convergence is obtained. The convergence criterion can be one of the following:

1. $\underset{i}{\text{Max}} \left|x_i^{(k+1)} - x_i^{(k)}\right| < \varepsilon_1 \quad i = 1, 2$
2. $\underset{i}{\text{Max}} \left|f_i\right| < \varepsilon_2 \quad i = 1, 2$ (A.34)

Values of ε_1 and ε_2 are chosen depending on the problem, accuracy desired, etc.

The above procedure for two equations can be easily generalized for a system of n equations of the form (A.1) repeated below for convenience.

$$\begin{aligned} f_1(x_1, x_2, \ldots, x_n) &= 0 \\ f_2(x_1, x_2, \ldots, x_n) &= 0 \\ &\vdots \\ f_n(x_1, x_2, \ldots, x_n) &= 0 \end{aligned} \quad \text{(A.35)}$$

In matrix notation these equations are denoted by ***F(X)*** = 0. Define the Jacobian matrix denoted by ***F(X)*** as follows:

$$F'(X) = \left(\frac{\partial f_i}{\partial xj}\right) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \quad \text{(A.36)}$$

Then the Newton-Raphson iterates are defined by

$$X^{(k+1)} = X^{(k)} - \left[F'(X)^{(k)}\right]^{-1} F(X)^{(k)} \quad \text{(A.37)}$$

Convergence can be tested either in terms of x_i or f_i as in Equation (A.34).

There are no strict guidelines as to which method to choose in a given problem. The Newton-Raphson method has superior convergence properties as compared to the Gauss-Seidel method, i.e., it takes fewer iterations to converge to a solution. This is true provided the initial guess is fairly close to the desired solution. The disadvantage of the Newton-Raphson method is that the first-order derivatives have to be computed. The Gauss-Seidel method is easier to program and no matrix inversion is involved.

In power system problems F′(X) in Equation (A.36) is sparse. Special techniques of programming which uses only the nonzero elements together with triangular factorization obviating the need to take inverse in Equation (A.37) are used. There are a number of books dealing with sparsity-based techniques. This combined with simplification of the Jacobian, as discussed in Chapter 6, makes this technique a preferred one in power system problems.

Matrix Factorization

If we have to solve ***x*** for a different set of vectors ***b***, then it will be convenient to ystematize the procedure in the preceding section. One such method to factorize the matrix ***A*** as ***A*** = ***LU*** or ***L′DU***.

LU Factorization

Let

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} \ell_{11} & 0 & 0 \\ \ell_{21} & \ell_{22} & \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \boldsymbol{LU} \quad \text{(A.38)}$$

Multiplying ***L*** and ***U*** and equating the elements with those of ***A*** gives

$$\begin{aligned} a_{11} &= \ell_{11} & a_{12} &= \ell_{11}u_{12} & a_{13} &= \ell_{11}u_{13} \\ a_{21} &= \ell_{21} & a_{22} &= \ell_{21}u_{12} + \ell_{22} & a_{23} &= \ell_{21}u_{13} + \ell_{22}u_{23} \\ a_{31} &= \ell_{31} & a_{32} &= \ell_{31}u_{12} + \ell_{32} & a_{33} &= \ell_{31}u_{13} + \ell_{32}u_{23} + \ell_{33} \end{aligned} \quad \text{(A.39)}$$

Solving for ℓ_{ij}''s and u_{ij}''s gives

$$\ell_{11} = a_{11} \qquad u_{12} = \frac{a_{12}}{\ell_{11}} = \frac{a_{12}}{a_{11}} = a_{12}^{(1)} \qquad u_{13} = \frac{a_{13}}{\ell_{11}} = \frac{a_{13}}{a_{11}} = a_{13}^{(1)}$$

$$\ell_{21} = a_{21} \qquad \ell_{22} = a_{22} - \ell_{21}u_{12} \qquad u_{23} = \frac{1}{\ell_{22}}\left(a_{23} - \ell_{21}u_{13}\right)$$

$$= a_{22} - a_{21}a_{12}^{(1)} \qquad = \frac{1}{a_{22}^{(1)}}\left(a_{23} - a_{21}a_{13}^{(1)}\right)$$

$$= a_{22}^{(1)} \qquad = \frac{a_{23}^{(1)}}{a_{22}^{(1)}} = a_{23}^{(2)} \qquad (6.105)$$

$$\ell_{31} = a_{31} \qquad \ell_{32} = a_{32} - \ell_{31}u_{12} \qquad \ell_{33} = a_{33} - \ell_{31}u_{13} - \ell_{32}u_{23}$$

$$= a_{32} - a_{31}a_{12}^{(1)} \qquad = a_{33} - a_{31}a_{13}^{(1)} - a_{32}^{(1)}a_{23}^{(1)}$$

$$= a_{32}^{(1)} \qquad = a_{33}^{(1)} - a_{32}^{(1)}a_{23}^{(2)}$$

$$= a_{33}^{(2)}$$

Thus,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22}^{(1)} & 0 \\ a_{31} & a_{32}^{(1)} & a_{33}^{(2)} \end{bmatrix} \begin{bmatrix} 1 & a_{12}^{(1)} & a_{13}^{(1)} \\ 0 & 1 & a_{23}^{(2)} \\ 0 & 0 & 1 \end{bmatrix} \qquad (A.40)$$

i.e., $\boldsymbol{A} = \boldsymbol{LU}$. Note that ℓ_{ij}'s and u_{ij}'s are also obtained during the Gaussian elimination. Thus the $\boldsymbol{LU}$ factors are derivable from the Gaussian elimination process. We illustrate this by means of an example.

Example A.1 Using Gaussian elimination, write $\boldsymbol{A} = \boldsymbol{LU}$ for the $\boldsymbol{A}$ matrix in Example 6.3.

Solution:

$$[A] = \begin{bmatrix} \underline{2} & 1 & 3 \\ \underline{2} & 3 & 4 \\ \underline{3} & 4 & 7 \end{bmatrix} \qquad [b] = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

We follow the same steps as in Section 6.5 on Gaussian elimination. The underlined elements are the elements of $\boldsymbol{L}$ and $\boldsymbol{U}$.

Step 1

$$x_1 + \frac{1}{\underline{2}}x_2 + \frac{3}{\underline{2}}x_3 = \frac{b_1}{2} = b_1^{(1)}$$

$$\underline{2}x_2 + x_3 = b_2 - 2\left(b_1^{(1)}\right) = b_2^{(1)}$$

$$\frac{5}{\underline{2}}x_2 + \frac{5}{2}x_3 = b_3 - 3\left(b_1^{(1)}\right) = b_3^{(1)} \qquad (A.41)$$

Step 2

$$x_1 + \frac{1}{2}x_2 + \frac{3}{2}x_3 = b_1^{(1)}$$

$$x_2 + \frac{1}{\underline{2}}x_3 = \frac{b_2^{(1)}}{2} = b_2^{(2)}$$

$$\frac{5}{\underline{4}}x_3 = b_3^{(1)} - \left(\frac{5}{2}\right)b_2^{(2)}$$

$$= b_3^{(2)} \qquad (A.42)$$

Step 3

$$x_1 + \frac{1}{2}x_2 + \frac{3}{2}x_3 = b_1^{(1)}$$

$$x_2 + \frac{x_3}{2} = b_2^{(2)}$$

$$x_3 = \frac{b_3^{(2)}}{5/4} = b_3^{(3)} \qquad (A.43)$$

The elements of $\boldsymbol{L}$ and $\boldsymbol{U}$ matrices are underlined. Therefore,

$$\mathrm{L} = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & \frac{5}{2} & \frac{5}{4} \end{bmatrix}, \quad \mathrm{U} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \qquad (A.44)$$

It can be verified that $\boldsymbol{LU} = \boldsymbol{A}$.

***L′DU* Factorization** — Suppose $\boldsymbol{L}$ is split up as $\boldsymbol{L'D}$ where

$\boldsymbol{D}$ = diagonal matrix whose entries are diagonal elements of $\boldsymbol{L}$

$\boldsymbol{L'}$ = lower triangular matrix with diagonal elements = 1

$\boldsymbol{L'}$ is obtained from $\boldsymbol{L}$ by dividing the ith column of $\boldsymbol{L}$ by the diagonal element of that column for the matrix

$$\mathrm{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad (A.45)$$

The $\boldsymbol{L'DU}$ decomposition is

$$\mathrm{L} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{a_{21}}{a_{11}} & 1 & 0 \\ \frac{a_{31}}{a_{11}} & \frac{a_{32}^{(1)}}{a_{22}^{(1)}} & 1 \end{bmatrix} \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22}^{(1)} & 0 \\ 0 & 0 & a_{33}^{(2)} \end{bmatrix} \begin{bmatrix} 1 & a_{12}^{(1)} & a_{13}^{(1)} \\ 0 & 1 & a_{23}^{(2)} \\ 0 & 0 & 1 \end{bmatrix} \qquad (A.46)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ a_{21}^{(1)} & 1 & 0 \\ a_{31}^{(1)} & a_{32}^{(2)} & 1 \end{bmatrix} \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22}^{(1)} & 0 \\ 0 & 0 & a_{33}^{(2)} \end{bmatrix} \begin{bmatrix} 1 & a_{12}^{(1)} & a_{13}^{(1)} \\ 0 & 1 & a_{23}^{(2)} \\ 0 & 0 & 1 \end{bmatrix} \qquad (A.47)$$

Thus for the **A** matrix of Example 6.3

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & \frac{5}{2} & \frac{5}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{3}{2} & \frac{5}{4} & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{5}{4} \end{bmatrix}$$

$$= \boldsymbol{L'D} \tag{A.48}$$

The advantage of this factorization is that if **A** is symmetric, than both **L′** and **U** are transposes of each other. Thus, in the computer we need store only **L′** or **U**. Recall that in the fast decoupled power flow method both **B′** and **B″** are both symmetric if there are no phase shifters. With this factorization we have

$$\boldsymbol{Ax} = b \tag{6.115}$$

$$\boldsymbol{L'DUx} = \boldsymbol{b} \tag{6.116}$$

which is equivalent to

$$\boldsymbol{Ux} = \boldsymbol{y} \tag{6.117}$$

$$\boldsymbol{Dy} = \boldsymbol{y'} \tag{6.118}$$

$$\boldsymbol{L'y'} = \boldsymbol{b} \tag{6.119}$$

We first solve for **y′**, then **y**, and finally **x**.

Partitioning of matrices

It is sometimes convenient from the point of view of computation, to divide the matrix into a number of lower order matrices called *submatrices*, which can be easily manipulated. It is also helpful in some instances to do such a division to separate one group of elements from another. Thus

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$$

Addition and subtraction of these matrices are similar to those for ordinary matrices. The submatrices must be conformable, i.e., they must be of appropriate order.

$$\begin{bmatrix} A_1 & \vdots & A_2 \\ \cdots & \vdots & \cdots \\ A_3 & \vdots & A_4 \end{bmatrix} \pm \begin{bmatrix} B_1 & \vdots & B_2 \\ \cdots & \vdots & \cdots \\ B_3 & \vdots & B_4 \end{bmatrix} = \begin{bmatrix} A_1 \pm B_1 & \vdots & A_2 \pm B_2 \\ \cdots\cdots\cdots\cdots & \cdots & \cdots\cdots\cdots\cdots \\ A_3 \pm B_3 & \vdots & A_4 \pm B_4 \end{bmatrix}$$

Multiplication

$$\begin{bmatrix} A_1 & \vdots & A_2 \\ \cdots & \vdots & \cdots \\ A_3 & \vdots & A_4 \end{bmatrix} \begin{bmatrix} B_1 & \vdots & B_2 \\ \cdots & \vdots & \cdots \\ B_3 & \vdots & B_4 \end{bmatrix} = \begin{bmatrix} A_1B_1 + A_2B_3 & \vdots & A_1B_2 + A_2B_4 \\ \cdots\cdots\cdots\cdots\cdots & \vdots & \cdots\cdots\cdots\cdots\cdots \\ A_3B_1 + A_4B_3 & \vdots & A_3B_2 + A_4B_4 \end{bmatrix}$$

It is evident that in addition to **A** and **B** being conformable (i.e., the number of columns of **A** = the number of rows of **B**), the number of columns of **A** to the left of the dotted vertical line must be equal to the number of rows of **B** above the horizontal dotted line.

Transpose — The transpose of a partitioned matrix

$$A = \begin{bmatrix} A_1 & \vdots & A_2 \\ \cdots & \vdots & \cdots \\ A_3 & \vdots & A_4 \end{bmatrix} \text{ is given by } A^T = \begin{bmatrix} A_1^T & \vdots & A_3^T \\ \cdots & \vdots & \cdots \\ A_2^T & \vdots & A_4^T \end{bmatrix}$$

Inverse of a partitioned matrix — This is useful in the computation of the inverse of a matrix **A** in terms of inverses of matrices of lower order. Let **A** be of order $n \times n$. Partition **A** into four submatrices $\boldsymbol{A}_{11}$, $\boldsymbol{A}_{12}$, $\boldsymbol{A}_{21}$, and $\boldsymbol{A}_{22}$ of dimension $r \times r$, $r \times s$, $s \times r$, and $s \times s$, respectively, such that $r + s = n$.

$$A = \begin{bmatrix} A_{11} & \vdots & A_{12} \\ \cdots & \vdots & \cdots \\ A_{21} & \vdots & A_{22} \end{bmatrix}$$

Inverse of **A** may be similarly partitioned

$$A^{-1} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$$

We wish to find $\boldsymbol{D}_{ij}$ in terms of $\boldsymbol{A}_{ij}$. Since $\boldsymbol{AA}^{-1} = \boldsymbol{U}$, we have

$$\boldsymbol{A}_{11}\boldsymbol{D}_{11} + \boldsymbol{A}_{12}\boldsymbol{D}_{21} = \boldsymbol{U}$$

$$\boldsymbol{A}_{11}\boldsymbol{D}_{12} + \boldsymbol{A}_{12}\boldsymbol{D}_{22} = 0$$

$$\boldsymbol{A}_{21}\boldsymbol{D}_{11} + \boldsymbol{A}_{22}\boldsymbol{D}_{21} = 0$$

$$\boldsymbol{A}_{21}\boldsymbol{D}_{12} + \boldsymbol{A}_{22}\boldsymbol{D}_{22} = \boldsymbol{U}$$

These are solved for $\boldsymbol{D}_{11}$, $\boldsymbol{D}_{12}$, $\boldsymbol{D}_{21}$, and $\boldsymbol{D}_{22}$. The best formulation involves a minimum number of matrix inversions. The minimum is 2 and the solution yields the following:

1. Compute A_{11}^{-1}
2. Compute $D_{22} = \left(A_{22} - A_{21}A_{11}^{-1}A_{12}\right)^{-1}$

$$D_{21} = -D_{22}A_{21}A_{11}^{-1}$$

$$D_{12} = -A_{11}^{-1}A_{12}D_{22}$$

$$D_{11} = -A_{11}^{-1} + A_{11}^{-1}A_{12}D_{22}A_{21}A_{11}^{-1}$$

Partitioning does not save time but is useful in inverting matrices that are too large to store in the main memory of the computer.