

Stabilization of a Chaotic System Using Fuzzy Parallel Distributed

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Abstract

In this article a fuzzy controller is designed for a chaotic model whose structure is represented by the Takagi-Sugeno model. Using the T-S model of chaotic systems, the control algorithm is developed based on parallel distributed compensator concept and the Lyapunov method [1] is used to carry out stability analysis of a real system structure. In terms of PDC fuzzy control theory, the whole control sub-system's fuzzy control can be achieved by each subsystem's control. The simulation results shows the effectiveness and feasibility of the method to stabilize a Lorenz system

Keyword: Liapanoph, Takagi-Sugeno Fuzzy Model, Parallel Compensator, Fuzzy Controller, Parallel Distributed Controller (PDC)

JEL Codes: D4, D21, E5, E42, 44.

Introduction

Control systems are increasingly required to have high dynamical performance and robust behaviours, yet are expected to cope with more complex, uncertain and highly nonlinear dynamic processes. As the complexity of process increases, abstraction and uncertainty in the models and their mathematical representation also increase. One significant approach in dealing with major changes and uncertainty in nonlinear dynamical processes is through intelligent modeling and control.

In some control tasks, the systems to be controlled have uncertainty parameter. Unless such parameter is gradually reduced online by an appropriate adaptation or estimation mechanism, it may cause inaccuracy or instability for the control systems.

Many nonlinear systems have been known to exhibit chaotic dynamics as well as period oscillations. A chaotic system is a nonlinear deterministic system which is very sensitive to small perturbation in its initial condition and its long time behaviour is unpredictable. Although the model description of some chaotic systems is simple, the dynamic behaviour is very complex. Since the research of Ott, Grebogi and Yorke (OGY) (Chen, 1999), many researchers have managed to use modern theories to control chaotic systems, most of them based on exact chaotic model.

Fuzzy control methods have advantages such as robustness, which have been demonstrated through industrial applications. Fuzzy controllers are supposed to work in situations where there is a large uncertainty or unknown variation in plant parameters and structures.

Takagi-Sugeno Fuzzy Model

Fuzzy logic controllers are generally considered applicable to plants that are mathematically poorly understood and where experienced human operators are available. However, fuzzy control has not been regarded as a rigorous science due to the lack of the guarantee of the global stability and acceptable performance. To overcome this drawback, as Takagi-Sugeno(T-S) fuzzy

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model which can express a highly nonlinear functional relation in spite of a small number of fuzzy implication rules was proposed in studies by Aracil and Gordillo (2000) and Farinwata, Filev and Langari (2000), there has been significant research on the stability analysis and systematic design of fuzzy controllers.

The Takagi-Sugeno type fuzzy model uses the smooth aggregation of local linear mathematical models to represent the dynamical systems, which are useful because they can provide description of a physical phenomenon or a process, and can be well suited to analysis, prediction and design of dynamic control system.

Consider the continuous-time nonlinear system described by the TS model. The rule of continuous-time TS model is of the following form:

Rule i

If $x_1(t) : M'_1 x_2(t) : M'_2 \dots x_p(t) : M'_p$

Then $X(t) = A1 X(t) + B1 U(t)$

$$i = 1, 2, \dots, l \tag{1}$$

Where

$$X(t) = [x_1, x_2, \dots, x_n] \tag{2}$$

Using Parallel Distributed Controller (PDC), we can construct the control rules as:

Rule i

If $x_1(t) : M'_1 x_2(t) : M'_2 \dots x_p(t) : M'_p$

Then $U(t) = K_1 X(t)$

$$i = 1, 2, \dots, l \tag{3}$$

By substituting the controller (3) into model (1), we can construct the closed-loop fuzzy control system as following:

$$X(t) = \frac{\sum_{i=1}^l (w_i (A_i X(t) + B_i U(t)))}{\sum_{i=1}^l w_i}$$

$$w_i = \prod_{j=1}^l M_j^I(x_j(t)) \tag{4}$$

A sufficient condition for ensuring the stability of the closed-loop fuzzy system is given as follows:

The equilibrium of a fuzzy control system (1) is asymptotically stable in the large if there exists a common positive definite matrix P such that

$$A_i^T P + P A_i < 0 \quad i = 1, 2, \dots, l \tag{5}$$

Rule i

If $x_1(t) : M'_1 x_2(t) : M'_2 \dots x_p(t) : M'_p$

Then $U(t) = K_1 X(t)$

$$i = 1, 2, \dots, l \tag{6}$$

The final system will be formulated as:

$$X(t) = \frac{\sum_{i=1}^l \sum_{j=1}^l w_i w_j (A_i X(t) + B_i K_j) X(t)}{\sum_{i=1}^l \sum_{j=1}^l w_i w_j} \tag{7}$$

Following theorem 1, the sufficient condition for the stability of the closed-loop is

$$(A_i + B_i K_j)^T P + P(A_i + B_i K_j) < 0$$

$$i, j = 1, 2, \dots, l \tag{8}$$

It should be emphasized that the stability criteria and the above inequality are LMIs, if and only if the feedback gains are pre-determined using the proper design. By changing the variables we will get the following conditions:

$$(A_i + B_i K_j)^T P + P(A_i + B_i K_j) < 0$$

$$i = 1, 2, \dots, l$$

$$G_{ij} = \frac{(A_i - B_i K_i) + A_i + B_i K_i}{2} < 0$$

$$i < j \leq l \tag{9}$$

As it is observed above we cannot use this method for finding the feedback gains of the systems with uncertainty. By changing the variables into $V_i = K_i Q, Q = P^{-1}$, we will get the following LMI equations [5].

$$Q A_i^T + A_i Q - V_i^T B_i^T - B_i V_i < 0$$

$$i = 1, 2, \dots, x$$

$$Q A_i^T + A_i Q + Q A_i^T + A_i Q - V_i^T B_i^T - B_i V_i - V_i^T B_i^T - B_i V_i < 0$$

$$i < j \leq r$$

$$STO \quad h_1 \cap h_1 = \phi \tag{10}$$

Simulation

In this section, the validity and effectiveness of the proposed fuzzy estimators are examined through the simulation of uncertain chaotic system control. A Lorenz

system is chosen to demonstrate the ability of the proposed scheme. The uncontrolled model for a Lorenz system is given by:

$$\begin{aligned} x_1'(t) &= -ax_1(t) + ax_2(t) + u(t) \\ x_2'(t) &= cx_1(t) - x_2(t) - x_1(t)x_3(t) \\ x_3'(t) &= x_1(t)x_2(t) - b_3(t) \end{aligned} \quad (11)$$

The Lorenz model is used for the fluid convection description, especially for some features of the atmospheric dynamics (Feng and Chen, 2005). x_1 , x_2 , and x_3 represent measures of fluid velocity, and horizontal and vertical temperature variations, respectively. a , c , and b are positive parameters representing the Prandtl number, Rayleigh number and geometric factor, respectively. In these simulations, we will consider the system with $a = 10$, $b = 8/3$, $c = 28$. On the other hand, solving the equitation, $(X(t) = f(X, t) = 0)$ equilibrium points of "Lorenz" equation is obtained as below:

$$\begin{aligned} X(t) &= [-\sqrt{b(r-1)} - \sqrt{b(r-1)} r - 1]^T \\ X(t) &= [+ \sqrt{b(r-1)} + \sqrt{b(r-1)} r - 1]^T \\ X(t) &= [0 \ 0 \ 0]^T \end{aligned} \quad (12)$$

By taking Jakobin matrix special values into account, the equilibrium points status of the system is determined to be nonlinear

$$A = \frac{\partial f}{\partial x_{x=\bar{x}}} = \begin{bmatrix} -\sigma & \sigma & 0 \\ r - \bar{x}_3 & -1 & -\bar{x}_1 \\ \bar{x}_2 & \bar{x}_1 & -b \end{bmatrix} \quad (13)$$

The special value of the above matrix for quiescent point $X(t) = [0 \ 0 \ 0]^T$ is:

$$\begin{aligned} \lambda_{1,2} &= -\frac{\sigma+1}{2} \pm \frac{1}{2} \sqrt{(\sigma+1)^2 + 4\sigma(r-1)} \\ \lambda_3 &= -b \end{aligned} \quad (14)$$

As it is observed for $r = 1$, the real part of special

$$\begin{aligned} A_1 &= \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -d \\ 0 & d & -\frac{8}{3} \end{bmatrix} \\ A_2 &= \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -d \\ 0 & -d & -\frac{8}{3} \end{bmatrix} \end{aligned}$$

Values will be equal to zero. The special values of the above matrix for quiescent point

$$\begin{aligned} X(t) &= [+ \sqrt{b(r-1)} + \sqrt{b(r-1)} r - 1]^T \\ X(t) &= [- \sqrt{b(r-1)} - \sqrt{b(r-1)} r - 1]^T \end{aligned} \quad (15)$$

are the roots of characteristic equation

$$S^3 + (1 + \sigma + b)s^2 + b(\sigma + r)s + 2\sigma b(r - 1) = 0 \quad (16)$$

In order to apply the suggested fuzzy controller, we need a T-S fuzzy model representation of chaotic systems. An exact fuzzy modeling has been employed to construct the fuzzy models for the Lorenz system. It utilizes the concept of sector nonlinearity. The following fuzzy model exactly represents the nonlinear equation of the Lorenz system under the assumption that $x_1(t) \in [-d, d]$

$$\begin{aligned} \max(x_1(t)) (M_{11}(x_1(t)) + \min(x_1(t)) * M_{21}(x_1(t))) \\ = x_1(t) \\ M_{11}(x_1(t)) + M_{21}(x_1(t)) = 1 \end{aligned} \quad (17)$$

Hence solving equations:

$$\begin{aligned} d * M_{11}(x_1(t)) - d * M_{21}(x_1(t)) = x_1(t) \\ M_{11}(x_1(t)) + M_{21}(x_1(t)) = 1 \end{aligned} \quad (18)$$

The suitable phase membership function will be obtained as below:

$$\begin{aligned} M_{11} = (x_1(t)) &= \frac{1}{2} \left(1 + \frac{x_1(t)}{d} \right) \\ M_{21} = (x_1(t)) &= \frac{1}{2} \left(1 - \frac{x_1(t)}{d} \right) \end{aligned} \quad (19)$$

The fuzzy model of the Lorenz system will be

Rule 1:

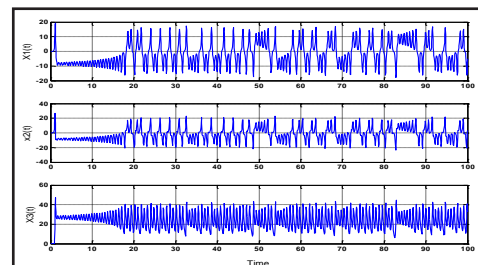
$$\text{If } x_1(t) \text{ is } M_{11} \text{ Then } X(t) = A_1 X(t) + B_1 U(t)$$

Rule 2:

$$\text{If } x_1(t) \text{ is } M_{12} \text{ Then } X(t) = A_1 X(t) + B_1 U(t) \quad (20)$$

$$B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (21)$$

Fig. 1: State Vectors of the Uncontrolled Process



Control Design Based on Wide Parallel Compensator

In this section, using wide parallel compensation method, we design the control signal so that stability conditions mentioned in earlier section can be provided. Using $V_i = K_i Q$, $Q = P^{-1}$ Liapanoph matrix and feedback vector coefficients will be obtained as below:

$$K_1 = [-8.2035 \quad 216.9759 \quad -1.5693]$$

$$K_2 = [-8.2035 \quad 216.9759 \quad 1.5693] \quad (22)$$

By implementing feedback gain vector K_1 , K_2 the poles of closed loop system will be equal to:

$$-2.3528, -81.822i - 1.5551, 818220i - 1.5551$$

Considering each of Sugeno rules as an independent sub-system, the phase model of controller will be as below:

Control Rule 1:

$$\text{If } x_1(t) \text{ is } M_{11} \text{ Then } U(t) = -K_1 X(t) \quad (23)$$

Control Rule 2:

$$\text{If } x_1(t) \text{ is } M_{21} \text{ Then } U(t) = -K_2 X(t) \quad (24)$$

As it was mentioned before, for the system to be stable, it is necessary that there will be positive V_1, V_2 so that the matrix non-equation.

$$QA_i^T + A_i Q - V_i^T B_i^T - B_i V_i < 0$$

$$i = 1, 2$$

$$QA_1^T + A_1 Q + QA_2^T + A_2 Q - V_2^T B_1^T - B_1 V_2 - V_1^T B_2^T - B_2 V_1 < 0$$

$$STO \quad h_1 \cap h_1 = \phi \quad (25)$$

are established.

Using tool box LMI in MATLAB environment, the Q, V_1, V_2 values will be obtained as below

$$Q = \begin{bmatrix} 5.7528e-001 & 3.0040e-002 & -1.5582e-017 \\ 3.0040e-002 & 4.2495e-000 & 1.5440e-018 \\ -1.5582e-017 & 1.5440e-018 & 4.2554e-000 \end{bmatrix}$$

$$V_1 = [-16.9326 \quad 51.1794 \quad -0.3688]$$

$$V_2 = [-16.9326 \quad 51.1794 \quad 0.3688] \quad (26)$$

$$P = \begin{bmatrix} 5.758e-001 & 3.0040e-002 & -1.5582e-017 \\ 3.0040e-002 & 4.2495e-000 & 1.5440e-018 \\ -1.5582e-017 & 1.5440e-018 & 4.2554e-000 \end{bmatrix}$$

Fig.1 indicates the results of system simulation with primary conditions and calculated feedback coefficients.

Fig. 2: State Vectors of the Controlled Process

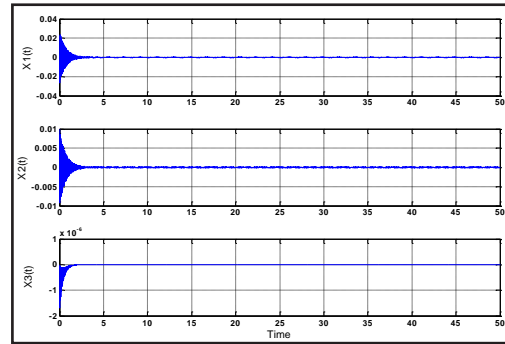
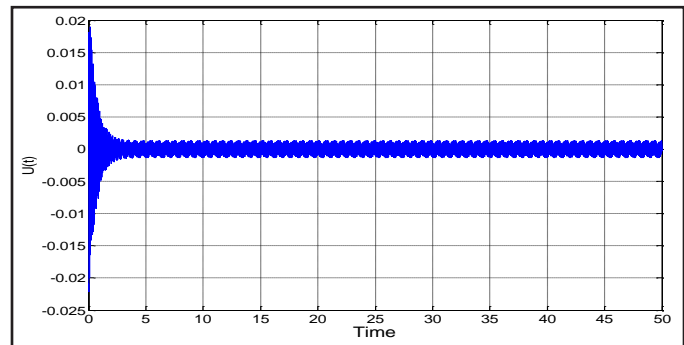


Fig. 3: Control Signal



In order to have a more reliable controller we consider another stabilisation condition to improve the performance of the system. Suppose we require limiting the control signal and considering the real parts of the poles to be less than -5.

Using tool box LMI in MATLAB environment, we get

$$P = \begin{bmatrix} 0.0902 & 0.0096 & -0.0036 \\ 0.0096 & 0.0497 & 0.0024 \\ -0.0036 & 0.0024 & 0.0513 \end{bmatrix}$$

The closed loop system poles will be

$$K_1 = [1.6600 \quad 21.6150 \quad 6.1368]$$

$$K_2 = [1.6600 \quad 21.6150 \quad 6.1368]$$

$$-5.1043 \pm 3.4530e + 001i, -5.1240$$

Fig. 4 shows the state vector and control signal with second controller.

Fig. 4: State Vectors of the Controlled Process with Two Controllers

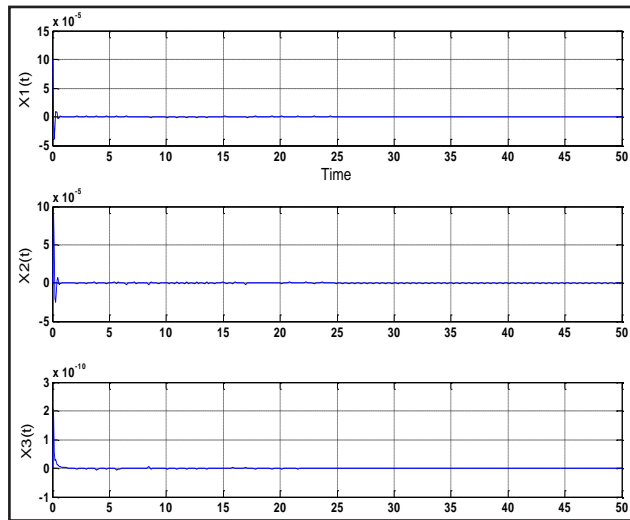
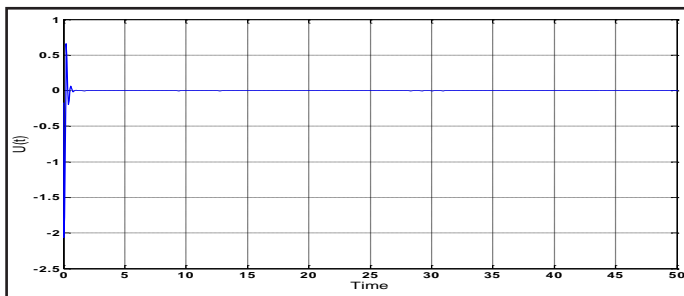


Fig. 5: Control Signal



Conclusion

In this paper the stabilizing controller for the fuzzy system with known structure is designed. The fuzzy controller parameters are determined according to the fuzzy model via PDC. The control algorithm is developed based on parallel distributed compensator concept and the Lyapunov method. In order to have a more reliable system, the control signal is limited and the poles of the system are designed to be located in a restricted space. The results show that the performance of the second fuzzy controller is better in comparison with the first one. As per the simulation results for the proposed fuzzy controller, the states go through zero with an acceptable convergence rate and the oscillations of control signal is much smaller.

References

- Aracil, J. & Gordillo, F. (2000). *Stability Issues in Fuzzy Control*. Springer Verlag.
- Chen, G. (1999). *Controlling Chaos and Bifurcations in Engineering Systems*. USA: CRC Press LLC.
- Farinwata, S. S., Filev, D. & Langari, R. (2000). *Fuzzy Control Synthesis and Analysis*. John Wiley.
- Feng, G. & Chen, G. (2005). Adaptive Control of Discrete-Time Chaotic Systems: A Fuzzy Control Approach. *Chaos, Solitons and Fractals*, 23(2), 459-467.
- Tanaka, K. & Wanh, H. O. (2001). *Nonlinear Analysis of Fuzzy Control Systems Design and Analysis*. John Wiley.