Robust Benchmarking of Indian Mutual Funds-A Partial Frontier Approach

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Abstract

Performance analysis of mutual funds is usually made on the basis of return-risk framework. Traditionally, excess return (over risk-free rate) to risk ratios were used for the purpose mutual fund evaluation. Subsequently, the application of non-parametric mathematical programming techniques in the context of performance evaluation facilitated multi-criteria decision making. However,the estimates of performance on the basis of conventional programming techniques like DEA and FDH are affected by the presence of outliers in the sample observations. The present, accordingly uses more robust benchmarking techniques for evaluating the performance od sectoral mutual fund schemes based on observations for the second half of 2010. The USP of the present study is that it uses two partial frontier techniques (Order-m and Order- α) which are less susceptible to the problem of extreme data.

Keywords: Mutual Fund, Robust Benchmarking, Order-m, Order- α, Partial Frontier Approach

Introduction

Mutual fund performance in financial markets are subject to market risks. Thus performance benchmarking of mutual funds requires the design of a return-risk framework against the backdrop of which fund performance is evaluated. Traditionally such evaluation was done in terms of excess return to risk/volatility ratios. In recent times, however, sophisticated frontier methods have been used for the purpose of mutual fund performance evaluation.

Against this backdrop, the present study discusses the application of non-parametric frontier method for benchmarking the performance of 16 best performing sectoral mutual fund schemes operating in the Indian market. The study shows how by using partial frontier approaches, it is possible to generate performance frontier of mutual funds without requiring knowledge about exogenous benchmark returns. The USP of the present study is that it provides robust estimation of efficiency which is less susceptible to extreme data.

Organisation of the Paper

The paper is organized in to six sections and proceeds as follows. Section 1 provides an overview of the Indian mutual fund industry. Section 2 provides a brief recap of the efficiency criteria developed on the basis of modern portfolio theory. Section 3 provides a brief discussion of the received literature on mutual fund benchmarking. Section 4 provides a discussion of the concept of full vis a vis partial frontier approach and introduces the concept of bootstrap. Section 5 outlines the conceptual framework and states the results. Finally, section 6 concludes.

Section 1 : Introduction to the Indian Mutual Fund Industry

The mutual fund industry in India started in 1963 with the formation of Unit Trust of India, at the initiative of the Government of India and Reserve Bank. Up to 1987, Unit Trust of India, government promoted institution had monopoly over the mutual fund market. There after competition was allowed in the industry to a limited extent as the government of India allowed the operation of public sector commercial bank (and insurance company) sponsored funds to operate in the mutual fund market. In 1987, two large public sector banks, namely, the State Bank of India and the Canara Bank floated mutual funds. They were followed by Life Insurance Corporation of India (June 1989), Punjab National Bank Mutual Fund (Aug 89), Indian Bank Mutual Fund (Nov 89), Bank of India (Jun 90), General Insurance Corporation of India

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Table 1: Mutual Fund Business in India: Growth in Asset Under Management

Particulars	December 1988	December 1993	September 2004	September 2012
Total Asset Under Management (Rs Million)	67000	4,70,004	15,31,080	74,03,017

(engaged in non-life business) and Bank of Baroda Mutual Fund (Oct 92).

In 1993, the mutual fund market was truly deregulated by allowing private and foreign competition. At the same time, the mutual funds came under the control of Securities Board of Exchange India- the capital market regulator. The permission granted for the private sector to operate in the Industry led to the introduction of a significant number of new players. Currently there are 40+ players in the market with Asset Under management exceeding Rs 74,00,000 Millions.

In the recent years, Sectoral Funds have gained popularity among investors because of high returns generated by them during bullish periods. Sectoral mutual fund schemes restrict their investment to specific high growth sectors of the economy and consequently are more risky than diversified equity funds.

Section 2 : Modern portfolio theory and portfolio benchmarking

The Mean-Variance Criteria

One of the earliest attempt towards portfolio benchmarking was by Markowitz (1952) and Tobin(1958) in the form of the mean-variance criterion.The basic idea behind the mean-variance approach is that the optimal portfolio for an investor is not simply any collection of securities but a balanced portfolio which provides the investor with the best combination of return and risk where return is measured by the expected value and risk is measured by the variance of the probability distribution of portfolio return.The decision rule which emanates from the M-V efficiency criteria is as follows:Given two discrete return distributions f(x) and g(x) , investors will prefer F(x) over F(G) if $\mu_F \geq \mu_G$ and $Var_F \leq Var_G$ (not both equalities holding simultaneously).

Markowitz pointed out that in the context of risk aversion, a quadratic of the form $a+bR+cR^2$ provides a close approximation of a smooth and concave utility function. In this case, maximization of expected utility implies:

$Max\ E[U(R)] = Max\ [a + b\mu + c\ E(R^2)] = Max\ [a + b\ \mu + c\ (\mu^2+\sigma^2)]$

Where μ =expected value of R and σ^2 = variance of R. Therefore, this investor will choose his portfolio solely on the basis of the mean and variance of R.

Stochastic Dominance Based Efficiency Criteria:

While Markowitz derived the optimal portfolio frontier with variance/standard deviation as a measure of risk, Roy (1952) conceptualized an investor who would be guided by the safety of principal first when dealing with risk and his contribution was instrumental in the development of downside measures of risk. He stated that the investor would prefer the investment with the smallest probability of going below the target return. By maximizing a reward to variability ratio, $(r_e - t)/s$, the investor will choose the portfolio with the lowest probability of going below the target level, t, given a expected mean return, r_e, and a standard deviation, s. Subsequently, Hadar and Russell (1969) pointed out that excepting some special cases (like the quadratic utility function), the specification of distributions in terms of their moments is not likely to yield strong results as information about the moments can not be used efficiently for the purpose of ordering uncertain prospects in a situation where the utility function is unknown. In this context, Hadar and Russell (1969) and Whitmore (1970) proposed three decision rules based on stochastic dominance(ordering) which are stronger than the moment method (see Appendix 1 for the decision rules).

Section 3 : Mutual Fund Benchmarking-the received literature

One of the earliest non-parametric study on mutual fund industry was by Murthi, Choi, and Desai (1997) who constructed a DEA portfolio efficiency index (DPEI), with standard deviation and transaction loads as inputs, and excess return as output, to investigate performance of 2083 mutual funds in the third quarter of 1993.In the first phase of empirical analysis, they compared the

DPEI measure with traditional measures of performance corresponding to 731 mutual funds belonging to seven categories: aggressive growth, asset allocation, equity-income, growth, growth-income, balanced and income. In the second phase, they used all 2083 mutual fund for computing DPEI for each fund. They also used a regression analysis to test for the source of variation in efficiency.

Sing and Ong (2000) applied a downside risk optimisation model for deriving the optimum portfolio in the context of financial market of Singapore for the period 1983 to 1997.

Basso and Funari (2001) in their non-parametric model, used several risk measures (standard deviation, standard semi-deviation and beta) and subscription and redemption costs as inputs, and the mean return and the fraction of periods in which the mutual fund was non-dominated as outputs. They used two DEA measures for the evaluation of performance. In the first measure, mutual fund return has been considered as the output and standard deviation and transaction cost have been taken as the inputs. In the second DEA measure, they developed a stochastic dominance indicator reflecting both the investors' preference structure and the time occurrence of returns. The model assigned a higher score to mutual funds which are not dominated by other mutual funds in the higher number of sub-periods. In a subsequent study (2003), Basso and Funari used an ethical score of mutual funds in place of the stochastic dominance indicator for the purpose of efficiency evaluation.

Gregoriou, Sedzro and Zhu (2005) applied DEA to appraise the performance of 168 hedge funds for the period 1997-2001.They initially used the Banker-Charnes-Cooper model to classify the hedge funds into efficient and inefficient categories. Thereafter, they used cross efficiency and super-efficiency models to further analyse the efficiency of funds.

Daraio and Simar (2006) evaluated performance of six categories of mutual funds (asset allocation, aggressive growth, balanced, equity income, growth and growth income) in terms of conditional input oriented order-m efficiency, Free Disposal Hull (FDH) method and DEA, Jensen's α and Sharpe Index. Total return has been taken as the output in the study while Expense Ratio, Loads and Turnover Ratio have been taken as the inputs. The study also compared the simple traditional ratio based indicators(Jensen's α and Sharpe Index) with their non-parametric counterparts (order m efficiency, DEA and FDH) using the Pearson, Spearman and Kendall's tau-b measures of correlation. The results indicate that while indicators based on nonparametric and robust approaches (DEA, FDH, order-m) are highly positively correlated, they are weakly correlated with the traditional indicators (Sharpe ratio and Jensen's alpha).

Zhao, Wang and Lai (2011) proposed two quadratic-constrained DEA models for endogenous performance benchmarking of mutual funds. They decomposed two vital factors for mutual funds performance, i.e. risk and return, in order to define mutual funds' endogenous benchmarks and give insights and suggestions for managements. Of the two quadratic-constrained DEA models used by them, one is a partly controllable quadratic-constrained programming. The approach is illustrated by a sample of twenty-five actual mutual funds operating in the Chinese Market corresponding to the years 2005 and 2006. The results show that although the market environment in year 2006 was much better than that in 2005, average efficiency score declined in year 2006 due to relaxing of system risk control. The majority of mutual funds do not show persistence in efficiency ranking. The results indicate that mutual fund ranking in China depends mostly on system risk controls.

Section 4: Methods of Portfolio Benchmarking

The Distance Function Approach

In the context of multi-criteria performance evaluation, Shephard's (1953,1970) distance function approach provides a sound conceptual basis for the derivation of evaluation criteria. The idea is invoked from a multi-input multi-output production system where distance function provide a functional characterisation of the structure of production technology. The input set of the production technology is characterised by the input distance function while the output set is characterised by the output distance function. The efficiency of a productive unit may be defined as a distance between the quantity of observed input and output and the quantity of input and output corresponding to the best practice frontier.

In order to elaborate the concept of distance function, we consider a technology T using a nonnegative vector of inputs $X=(x_1,x_2,......,x_n) \in R^n_+$ to produce a nonnegative vector of outputs $Y=(y_1,y_2,......,y_m) \in R^m_+$. In functional terms, they can be related as: Y=P(X) and X=L(Y)

Given this, an input distance function can be defined as $D_{input}= Max[\lambda:X/\lambda \in L(Y)]$. Intuitively speaking, an input distance function gives the maximum amount by which the producer's input vector can be radially contracted and yet remain feasible for the output vector it produces.The reciprocal of the input distance function can be considered as the radial measure of input oriented technical efficiency.

In an analogous fashion, the output distance function is defined as: $D_{output} = Min[\mu:Y/\mu \in P(X)]$. Intuitively speaking, an output distance function gives the minimum amount by which the producer's output vector can be deflated and yet remain feasible for a given input vector . The radial measure of output oriented technical efficiency coincides with the output distance function.

Comparison of performance using full frontier approaches

The performance of a mutual fund can be evaluated using Data Envelopment Analysis (DEA) and Free Disposal Hull approaches in the context of a return-risk framework without requiring knowledge about either the risk free rate and return on market portfolio. Data Envelopment Analysis is a non-parametric mathematical programming tool often used for comparing the relative performances of economic units with minimal prior assumption on input-output relation. The DEA approach constructs the efficiency frontier out of piecewise linear stretches thereby forming a convex production possibility set. The DEA method is a generalisation of Farrell's Single input single output technical efficiency measure to the multiple output- multiple input case. The methodology was originally developed by Charnes, Cooper and Rhodes (1978) and was later further extended by Banker, Charnes and Cooper (1984).

The Free Disposal Hull approach was developed by Deprins, Simar and Tulkens (1984). Like the DEA approach, FDH retains the free disposability of input and outputs. However, unlike the DEA approach, the FDH frontier is not convex. The distinguishing feature of the FDH approach is that efficiency evaluations are based on actual observations only and not on the convex combinations of actual observations.

Partial Frontier Approaches for Efficiency Evaluation

The full frontier approaches like DEA/FDH are susceptible to extreme data/outliers. This led to the introduction of two partial frontier approaches: Order-m and Order-alpha. The two approaches are now discussed in brief.

(a) Order-m Approach: The Order-m approach was introduced by Cazals, Florens and Simar (2002). In the input oriented Order-m approach, a decision making unit (x_0,y_0) is benchmarked against the average minimal input used by m peers randomly drawn from the population of decision making units producing at least y_0. The Order-m lower boundary of the input vector X is defined as the expected value of the minimum of m random variables $X_1, X_2 ,….., X_m$ drawn from the distribution function of X. Thus mathematically speaking, $\varphi_{m(input)} =E[min(X_1, X_2 ,…..,X_m)]$. The value m represents the number of m potential firms (drawn randomly from the population of firms) producing at least the output level of y against which we want to benchmark the observed firm. As m goes to infinity, the Order-m frontier converges to the full frontier.

In a similar manner, in the output oriented approach, benchmarking is done against the average maximal output used by m peers randomly drawn from the population of decision making units using inputs less than or equal to x_0. The Order-m upper boundary of the output vector Y is defined as the expected value of the maximum of m random variables $Y_1, Y_2 ,…..,Y_m$ drawn from the distribution function of Y. Thus mathematically speaking, $\varphi_{m(output)} =E[max(Y_1, Y_2 ,…..,Y_m)]$.

(b) Order-α Approach: As an alternative to the Order-m frontier, Aragon, Daouia and Thomas-Agnan (2005) introduced the Order-α frontier which is based on conditional quantiles of an appropriate distribution associated with the production process. Here a productive firm (x,y) is benchmarked against 100α firms producing an output level $\geq$y and using an input level$\leq$ x. The value of α lies between 0 and 1. As α goes to 1, the Order- α frontier converges to the full frontier.

Statistical Properties of Non-Parametric Estimators

The mathematical programming based evaluation technique were generally considered as those which provided point estimates of efficiency without any statistical foundation or justification leading to scepticism about the technique. Banker (1993), however, provided a formal statistical foundation for DEA. He showed that DEA estimators of the best practice monotone increasing and concave production function would be maximum likelihood estimators if the deviation of actual output from the efficient output is regarded as a stochastic variable with a monotone decreasing probability density function. For a finite sample size, the best practice frontier estimator would lie below the theoretical frontier implying the existence of an upward bias in the constructed frontier. However, for large samples, the bias approaches zero. He further showed that The DEA estimators exhibit the desirable asymptotic property of consistency, and the asymptotic distribution of the DEA estimators of inefficiency deviations is identical to the true distribution of these deviations.

In practical application of DEA, statistical estimators of the frontier are obtained from finite samples. Consequently, the corresponding efficiency estimates are sensitive to the sampling variations of the obtained frontier. Korostelev et al. (1995a, 1995b) have shown that DEA estimators satisfy consistency property under very weak general conditions. However, the obtained rates of convergence are very slow.

Bootstrap Estimators

Efron (1979) introduced the concept of bootstrap which involves resampling from an original sample of data via computer-based simulations to obtain the sampling properties of random variables. The starting point of any bootstrap procedure is a sample of observed data $X = \{x_1, x_2, \ldots, x_n\}$ drawn randomly from some population with an unknown probability distribution f . The premise of the bootstrap method is that the random sample actually drawn "mimics" its parent population.

The sample statistic $\theta = \theta(X)$ computed from this state of observed values is merely an estimate of the corresponding population parameter $\theta = \theta(f)$. Since the researcher has access to only one sample rather than multiple samples drawn from the same population, it is not possible to get sampling distribution of the statistic. Under the circumstances, if one draws a random sample with replacement from the observed values in the original sample, it can be treated like a sample drawn from the underlying population.

The bootstrap method suggested by Efron (1979) involves drawing of sample (with replacement) directly from the observed data and is known as naive bootstrap. In this case the bootstrap sample is effectively drawn from a discrete population which fails to recognise the fact that the underlying population density function f is continuous. Simar and Wilson (1998) suggested that the problem could be overcome by resorting to smoothed bootstrap which involves resampling via a fitted model. The smoothed bootstrap methodology involves the use of Kernel estimators as weight functions. If we write the naive bootstrap sample as $X_{nbs} = \{x_1^*, x_2^*, \ldots, x_n^*\}$ and the smoothed bootstrap sample as $X_{sbs} = \{x_1^{**}, x_2^{**}, \ldots, x_n^{**}\}$ then the elements of the two are related to each other in the following manner: $x_i^{**} = x_i^* + h \in \sim f$, where h is the smoothing parameter for the density function while x_i^* and x_i^{**} represent the ith elements of the naive and smoothed bootstrap samples.

In case of bootstrapping, every time when we replicate the bootstrap sample, we get a different sample X**, we will also get a different estimate of $\theta^* = \theta(X^{**})$. Thus, we need to select a large number of bootstrap samples, B, in order to extract as many combinations of x_j (j = 1, 2, . . . , n) as possible. The steps followed in bootstrapping are briefly as follows:

(a) Compute the technical efficiency θ from the observed sample X.

(b) Select r^{th} (r = 1, 2, . . . ,B) independent bootstrap sample X^*_r, which consists of n data values drawn with replacement from the observed sample X. From this, compute the naïve bootstrap.

(c) Compute the statistic $\theta_{sb} = \theta(X^{**}_{sb})$ from the rth bootstrap sample X^{**}_b

(d) Construct pseudo-data from the smoothed bootstrap efficiency scores and compute technical efficiency

(e) Repeat steps (b),(c) and (d) a large number of times (say, N times).

(f) Calculate the average of the bootstrap estimate as the arithmetic mean (θ_e).

An alternative approach to bootstrap efficiency is by Lothgren and Tambour (1999a,1999b). The underlying data generation process model follows Lothgren and Tambour (1999a,1999b). The inputs are given by random radial deviations off the input set. Formally, the input-output observations are given by

$(x_0, y_0) = (x_0^f / e_0, y_0)$ (2)

Where x_0 and y_0 represent the observed input and output vector and x_0 is related to the unobserved efficient input vector x_0^f by the relation $x_0 = x_0^f / e_0$ where e_0 is the stochastic input oriented technical efficiency. The procedure in each bootstrap resample is to mimic the data generating process implied by (2). The bootstrap Monte Carlo algorithms followed here are as under:

(i) The observed input-output vectors are transformed as $(x0^*, y0) = (x0\,\hat{e}, y0)$. The transformed input vectors are efficient.

(ii) The efficiency measures from the vector of VRS efficiency estimates are resampled independently with replacement. Let $\theta_b^* = (\theta_{b1}^*, \theta_{b2}^*, ..., \theta_{bn}^*)$ represent the resampled technical efficiency measures.

(iii) Let the bootstrap pseudo cases be given by $(x_{b0}^*, y_{b0}^*{}_0)$ where $x^*_{b0} = x^*_0 / \theta_{b0}$.

(iv) Estimate the bootstrap input oriented efficiency measures under variable returns to scale.

(v) Repeat steps (ii) to (iv) B times (say) to generate bootstrapped efficiency measures for the in-sample firms.

In the present study, the Simar-Wilson homogenous bootstrap method has been followed.

4.3.2 Correction of Bias

One important objective for applying bootstrap analysis in the context of small samples is to get rid of the upward bias existing in the estimated frontier. The bias correction procedure is now spelt out in brief:

A measure of the accuracy of an estimator θ_e of the parameter θ is the bias measure $E(\hat{\theta})$-θ. The bias-corrected estimator is: $\theta_{bc} = \hat{\theta}$ -bias. In our case, we compute bias $=\theta_e - \theta$.

Thus the bias corrected estimated technical efficiency is :$\theta_{bc} = 2\hat{\theta} - \theta_e$

However, as Simar and Wilson (2000) pointed out, this bias correction might create additional noise and the sample variances of the bootstrap values (σ_{bs}^2) are to be calculated.Bias correction is to be made only if : bias/$\sigma_{bs} > \sqrt{3}$.

Section 5: Framework of the study and the results:

5.1 Inputs and outputs and model orientation:

In the present study we make use of two output indicators and one input indicator. As output indicator, we take mean daily return and mean upside potential relative to mean daily return of the observed mutual fund (an indicator of second order stochastic dominance). As input indictor, standard deviation of daily return is included. Thus the input-output correspondence can be described as:

Output [Mean Return, Mean Upside Potential]=f(Standard Deviation)

In the present study we have used three variants of model orientation: input, output and Graph Hyperbolic. In the input oriented approach, we compare the actual input usage with benchmark input usage (for a given level of output) while in the output oriented approach, we compare the actual output with benchmark output (for a given level of input). In the Graph Hyperbolic approach we assume as if the fund tries simultaneously to maximise output and minimize input.

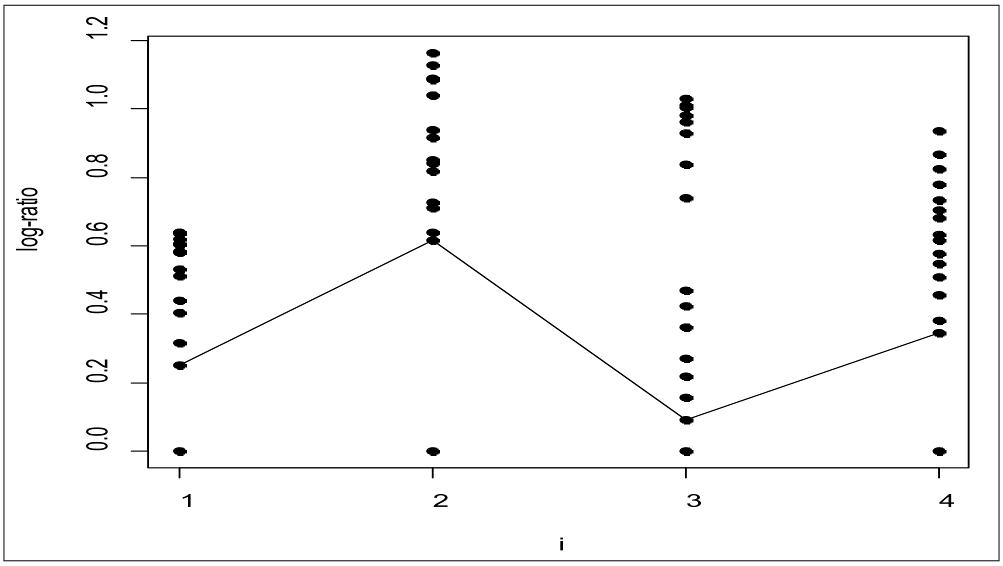
5.2 Period of Study:

The present study is based on observations relating to 16 sectoral mutual fund schemes for the period July 2010 to December 2010.The sectoral funds were selected on the basis of the annualized returns generated by them i.e. they were the top performing sectoral mutual funds for the period under consideration. The data have been collected from the AMFI (Association of Mutual Funds in India) website. Estimation has been made under the assumption that the funds operate under variable returns to scale.

5.3 Identification of Outlier

Using the approach suggested by Wilson, we first proceed to detect outliers from the sample by plotting log-ratio A line in the log-ratio plot connects the second smallest value of the ratios for each observation deleted to illustrate the separation between the smallest ratios for

Figure 1: Identification of Influential Observations



each observation. The plot is approximately linear under the homogeneity model. Under the heterogeneity model, the log-ratio plot shows convexity. In the present study, the resultant scenario is presented in figure 1.

5.4 Choice of m and alpha:

In the diagram presented below, we present the relationship between the value of m and alpha with the percentage of super-efficient firms included in the construction of the frontier. When alpha=1 the frontier is a full frontier where all super-efficient firma are included. Similarly, for m≥500, the order-m frontier also converges with the full frontier. On the basis of the relationship exhibited in figure-2 , the following values are selected: m=5 and alpha=0.5. The values correspond to a partial frontier where 12.5% of the funds are dropped.

Figure 2: Parameter- Outlier Relationship

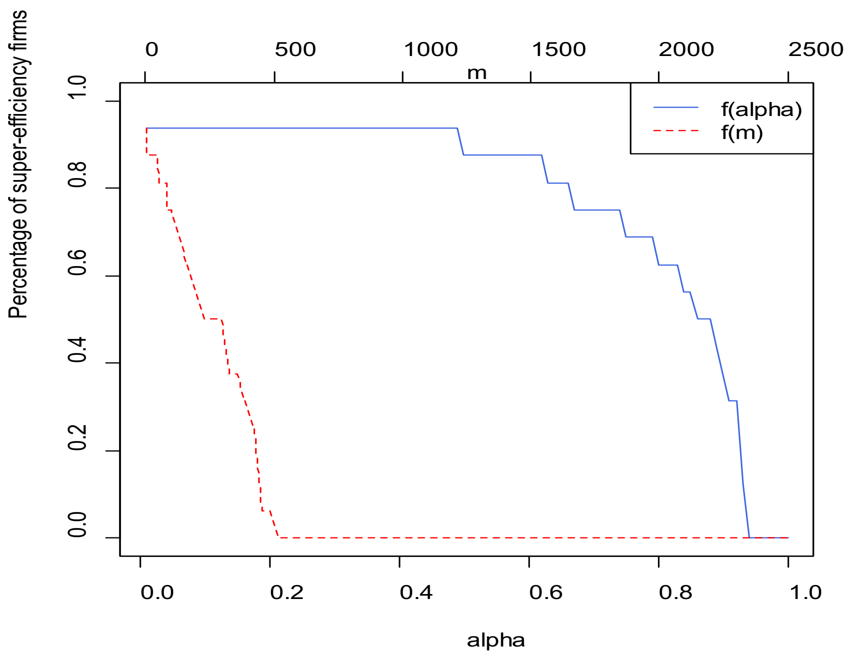


Table 2: Super-efficiency-partial frontier parameter relationship

Percentage of funds declared as super-efficient	*Value of m*	*Value of α*
87.5	5	0.50
75	85	0.67
50	228	0.86
25	420	0.92
0	516	1

Source: Calculated.

5.5 Descriptive Statistics of Technical Efficiency Scores:

Tables 2 and 3 present the descriptive statistics of technical efficiency scores corresponding to Order-m and Order-Alpha Frontier using three orientations- input oriented, output oriented and graph hyperbolic. The fund wise technical efficiency scores are provided in appendix tables A1 and A2.

Table 3: Descriptive Statistics of Efficiency Scores (Order-m)

Percentage of firms included	*Mean Technical Efficiency*		
	Input Oriented Approach	*Output Oriented Approach*	*Graph Hyperbolic Approach*
12.5	1.0851	1.0725	1.0951
25	0.9974	0.9982	0.9984
50	0.9973	0.9982	0.9982
75	0.9973	0.9982	0.9982
100	0.9973	0.9982	0.9982

Source: Calculated.

Table 4: Descriptive Statistics of Efficiency Scores (Order- α)

Percentage of firms included	*Mean Technical Efficiency*		
	Input Oriented Approach	*Output Oriented Approach*	*Graph Hyperbolic Approach*
12.5	1.4255	1.3059	1.4934
25	1.2364	1.1416	1.2484
50	1.0829	1.0509	1.0888
75	1.0105	1.0040	1.0489
100	0.9973	0.9982	0.9982

Source: Calculated.

5.6 Bootstrap Estimates

Computation procedure for bootstrap estimates of efficiency in case of Order- α is yet to be developed. Under the circumstances, we consider the bootstrap estimates in case of order-m. It is generally agreed that in case of order-m, re-sampling of data for 200 times provides robust bootstrap estimates. Table 5 provides the descriptive statistics of bootstrap estimates for different values of m.

Table 5: Bootstrap Estimates of Mean Technical Efficiency (Order-m)

Value of m	*Percentage of funds included in the frontier*	*Input Oriented Approach*	*Output Oriented Approach*	*Graph Hyperbolic Approach*
5	12.5	1.1164	1.0736	1.1066
85	25	0.9974	0.9982	0.9984
228	50	0.9973	0.9982	0.9984
420	75	0.9973	0.9982	0.9984
516	100	0.9973	0.9982	0.9984

Source: Calculated

Table 6: Bias Corrected Estimates of Mean Technical Efficiency (Order-m)

Value of m	*Percentage of funds included in the frontier*	*Input Oriented Approach*	*Output Oriented Approach*	*Graph Hyperbolic Approach*
5	12.5	1.0538	1.0714	1.0837
85	25	0.9974	0.9982	0.9984
228	50	0.9973	0.9982	0.9984
420	75	0.9973	0.9982	0.9984
516	100	0.9973	0.9982	0.9984

Source: Calculated.

Section 6: Conclusion

In India, the mutual fund industry has gained in terms of size and depth over the years in response to the introduction of more competition and the institution of global best practices in the matter of regulation/operation. Of late, sectoral funds are having growing popularity in the Indian market in view of the superior returns provided by them relative to diversified equity funds in a bullish market. Conventionally, the performance of mutual fund is evaluated in the context of return-risk framework. In the present study, two partial frontier methods have been used for the estimation of fund performance. The USP of both the techniques is that the researcher can make a trade off between sample coverage and robustness by choosing appropriate model parameters (m and α). Indeed, our study shows that robust performance evaluation is attained both in case of point and bootstrap estimates only considering 25% of the sample observations. Since, in a partial frontier approach, the entire data set is not considered, the estimates are less likely to affected by extreme data.

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Appendix 1: The concept of stochastic dominance

In order to provide a very brief introduction to the concept of stochastic dominance, let us consider a random variable x taking the values x_i. Let f and g denote the probability functions of x and $F(x_i)$ and $G(x_i)$ be the respective cumulative distributions. We now introduce the concept of First, Second and Third Order Stochastic Dominance.

(i) First Order Stochastic Dominance (FSD):

In our example elaborated above, f(x) dominates g(x) if $F(x) \leq G(x)$ for all x_i □ X. Hadar and Russell proved that under this rule distributions may be ordered according to preference under any utility functions.

(ii) Second Order Stochastic Dominance (SSD):

The second rule is weaker than the first rule. In the discrete case second order stochastic dominance implies that f(x) dominates g(x) if $\sum_r G(xi)\,\Delta x_i \leq \sum_r F(xi)\,\Delta x_i$ for all r<n where x_n is the largest value taken by the random variable and $\Delta x_i = x_{i+1} - x_i$. Under SSD, distributions may be ordered for any utility function which exhibits non-increasing marginal utility everywhere.

(iii) Third Order Stochastic Dominance

Whitmore(1970) introduced the concept of third degree stochastic dominance as follows: f(x) dominates g(x) if $\sum_r G(xi)(\Delta x_i)^2 \leq \sum_r F(xi)(\Delta x_i)^2$ for all r<n where x_n is the largest value taken by the random variable and $\Delta x_i = x_{i+1} - x_i$.

Appendix 2: Fund wise efficiency scores: (Order-m and Order-α)

Table A1: Order-m Efficiency Score (m=5)

Fund Name	Output Oriented Approach	Input Oriented Approach	Graph Hyperbolic Approach
Canara Robeco FORCE Fund - Institutional - Growth	1.0532	1.1425	1.1297
Canara Robeco FORCE Fund - Retail - Growth	1.1117	1.1463	1.1296
Franklin Infotech Fund - Growth	1.1905	1.0000	1.1186
Franklin Pharma Fund - Growth	1.0000	1.2373	1.1470
ICICI Prudential Banking and Financial Services Fund - Retail - Growth	1.0922	1.0000	1.0466
ICICI Prudential Technology Fund - Growth	1.3884	1.0000	1.1892
Reliance Banking Fund - Growth	1.1420	1.0000	1.1201
Reliance Pharma Fund-Growth Plan-Growth	1.0045	1.1313	1.0633
Religare Banking Fund - Regular - Growth	1.0467	1.0017	1.0397
Sahara Banking and Financial Services Fund - Growth	1.1460	1.0022	1.0308
Tata Life Sciences & Technology Fund - Growth	1.0069	1.1492	1.0929
Sundaram Financial Services Opportunities Fund - Retails - Growth	1.0683	1.0020	1.0448
UTI - MNC Fund (UGS 10000)-Growth Option	1.0027	1.1461	1.1042
UTI Banking Sector Fund - Growth	1.0688	1.0000	1.0688
UTI Pharma and Healthcare Fund - Growth	1.0516	1.1721	1.1445
UTI Services Industries Fund-Growth Option	0.9878	1.0293	1.0524

Source: Calculated.

Table A2: Order-m Efficiency Score (m=85)

Fund Name	Output Oriented Approach	Input Oriented Approach	Graph Hyperbolic Approach
Canara Robeco FORCE Fund - Institutional - Growth	1.0000	1.0000	1.0000
Canara Robeco FORCE Fund - Retail - Growth	1.0000	1.0000	1.0000
Franklin Infotech Fund - Growth	1.0000	1.0000	1.0002
Franklin Pharma Fund - Growth	1.0000	1.0001	1.0004
ICICI Prudential Banking and Financial Services Fund - Retail - Growth	1.0000	1.0000	1.0000
ICICI Prudential Technology Fund - Growth	1.0000	1.0000	1.0004
Reliance Banking Fund - Growth	1.0001	1.0000	1.0005
Reliance Pharma Fund-Growth Plan-Growth	1.0000	1.0000	1.0001
Religare Banking Fund - Regular - Growth	1.0000	1.0000	1.0000
Sahara Banking and Financial Services Fund - Growth	1.0000	1.0000	1.0000
Tata Life Sciences & Technology Fund - Growth	1.0000	1.0000	1.0002
Sundaram Financial Services Opportunities Fund - Retails - Growth	1.0000	1.0000	1.0001
UTI - MNC Fund (UGS 10000)-Growth Option	1.0000	1.0001	1.0002
UTI Banking Sector Fund - Growth	1.0002	1.0000	1.0002
UTI Pharma and Healthcare Fund - Growth	1.0000	1.0001	1.0006
UTI Services Industries Fund-Growth Option	0.9574	0.9712	0.9712

Source: Calculated.

Table A3: Order-m Efficiency Score (m=228,420,516)

Fund Name	Output Oriented Approach	Input Oriented Approach	Graph Hyperbolic Approach
Canara Robeco FORCE Fund - Institutional - Growth	1	1	1
Canara Robeco FORCE Fund - Retail - Growth	1	1	1
Franklin Infotech Fund - Growth	1	1	1
Franklin Pharma Fund - Growth	1	1	1
ICICI Prudential Banking and Financial Services Fund - Retail - Growth	1	1	1
ICICI Prudential Technology Fund - Growth	1	1	1
Reliance Banking Fund - Growth	1	1	1
Reliance Pharma Fund-Growth Plan-Growth	1	1	1
Religare Banking Fund - Regular - Growth	1	1	1
Sahara Banking and Financial Services Fund - Growth	1	1	1
Tata Life Sciences & Technology Fund - Growth	1	1	1
Sundaram Financial Services Opportunities Fund - Retails - Growth	1	1	1
UTI - MNC Fund (UGS 10000)-Growth Option	1	1	1
UTI Banking Sector Fund - Growth	1	1	1
UTI Pharma and Healthcare Fund - Growth	1	1	1
UTI Services Industries Fund-Growth Option	0.9574	0.9712	0.9712

Source: Calculated.

Table A4: Order- α Efficiency Score (α=0.5)

Fund Name	*Output Oriented Approach*	*Input Oriented Approach*	*Graph Hyperbolic Approach*
Canara Robeco FORCE Fund - Institutional - Growth	1.4524	1.4075	1.4343
Canara Robeco FORCE Fund - Retail - Growth	1.4546	1.4151	1.4420
Franklin Infotech Fund - Growth	2.0795	1.0000	1.7085
Franklin Pharma Fund - Growth	1.0000	1.9336	1.6162
ICICI Prudential Banking and Financial Services Fund - Retail - Growth	1.3919	1.0000	1.3898
ICICI Prudential Technology Fund - Growth	2.5292	1.0000	1.9127
Reliance Banking Fund - Growth	1.5811	1.0000	1.5787
Reliance Pharma Fund-Growth Plan-Growth	1.0123	1.5523	1.4314
Religare Banking Fund - Regular - Growth	1.3808	1.0556	1.3808
Sahara Banking and Financial Services Fund - Growth	1.7424	1.0210	1.3490
Tata Life Sciences & Technology Fund - Growth	1.0524	1.7409	1.4552
Sundaram Financial Services Opportunities Fund - Retails - Growth	1.3793	1.0654	1.3793
UTI - MNC Fund (UGS 10000)-Growth Option	1.0000	1.6725	1.5423
UTI Banking Sector Fund - Growth	1.3208	1.0000	1.4501
UTI Pharma and Healthcare Fund - Growth	1.1951	1.6582	1.5032
UTI Services Industries Fund-Growth Option	1.2366	1.3717	1.3216

Source: Calculated.

Table A5: Order- α Efficiency Score (α=0.67)

Fund Name	*Output Oriented Approach*	*Input Oriented Approach*	*Graph Hyperbolic Approach*
Canara Robeco FORCE Fund - Institutional - Growth	1.2916	1.2735	1.4048
Canara Robeco FORCE Fund - Retail - Growth	1.2779	1.2803	1.4124
Franklin Infotech Fund - Growth	1.7111	1.0000	1.2503
Franklin Pharma Fund - Growth	1.0000	1.3764	1.3764
ICICI Prudential Banking and Financial Services Fund - Retail - Growth	1.2377	1.0000	1.1705
ICICI Prudential Technology Fund - Growth	1.9156	1.0000	1.3998
Reliance Banking Fund - Growth	1.3326	1.0000	1.1764
Reliance Pharma Fund-Growth Plan-Growth	1.0123	1.2484	1.2124
Religare Banking Fund - Regular - Growth	1.1328	1.0000	1.1328
Sahara Banking and Financial Services Fund - Growth	1.3490	1.0019	1.0878
Tata Life Sciences & Technology Fund - Growth	1.0000	1.2392	1.2392
Sundaram Financial Services Opportunities Fund - Retails - Growth	1.1881	1.0000	1.1534
UTI - MNC Fund (UGS 10000)-Growth Option	1.0000	1.3134	1.3063
UTI Banking Sector Fund - Growth	1.1654	1.0000	1.1654
UTI Pharma and Healthcare Fund - Growth	1.1681	1.3861	1.2458
UTI Services Industries Fund-Growth Option	1.0000	1.1466	1.2405

Source: Calculated.

Table A6 : Order- α Efficiency Score (α=0.86)

Fund Name	*Output Oriented Approach*	*Input Oriented Approach*	*Graph Hyperbolic Approach*
Canara Robeco FORCE Fund - Institutional - Growth	1.0000	1.1743	1.1743
Canara Robeco FORCE Fund - Retail - Growth	1.0000	1.1806	1.1806
Franklin Infotech Fund - Growth	1.0574	1.0000	1.1053
Franklin Pharma Fund - Growth	1.0000	1.1661	1.1467
ICICI Prudential Banking and Financial Services Fund - Retail - Growth	1.0085	1.0000	1.0190
ICICI Prudential Technology Fund - Growth	1.9127	1.0000	1.2116
Reliance Banking Fund - Growth	1.1359	1.0000	1.1434
Reliance Pharma Fund-Growth Plan-Growth	1.0000	1.0327	1.0327
Religare Banking Fund - Regular - Growth	1.0161	1.0000	1.0161
Sahara Banking and Financial Services Fund - Growth	1.1621	1.0000	1.0019
Tata Life Sciences & Technology Fund - Growth	1.0000	1.0498	1.0524
Sundaram Financial Services Opportunities Fund - Retails - Growth	1.0150	1.0000	1.0182
UTI - MNC Fund (UGS 10000)-Growth Option	1.0000	1.0599	1.0930
UTI Banking Sector Fund - Growth	1.0514	1.0000	1.0514
UTI Pharma and Healthcare Fund - Growth	1.0000	1.1741	1.1741
UTI Services Industries Fund-Growth Option	0.9677	0.9764	1.0000

Source: Calculated.

Table A7 : Order- α Efficiency Score (α=0.92)

Fund Name	*Output Oriented Approach*	*Input Oriented Approach*	*Graph Hyperbolic Approach*
Canara Robeco FORCE Fund - Institutional - Growth	1	1	1.0107
Canara Robeco FORCE Fund - Retail - Growth	1	1	1.0054
Franklin Infotech Fund - Growth	1	1	1.0574
Franklin Pharma Fund - Growth	1	1	1.1107
ICICI Prudential Banking and Financial Services Fund - Retail - Growth	1.0085	1	1.0085
ICICI Prudential Technology Fund - Growth	1	1	1.1195
Reliance Banking Fund - Growth	1.1359	1	1.1359
Reliance Pharma Fund-Growth Plan-Growth	1	1.0327	1.0123
Religare Banking Fund - Regular - Growth	1.0011	1	1.0011
Sahara Banking and Financial Services Fund - Growth	1	1	1.0019
Tata Life Sciences & Technology Fund - Growth	1	1	1.0498
Sundaram Financial Services Opportunities Fund - Retails - Growth	1.0150	1	1.0150
UTI - MNC Fund (UGS 10000)-Growth Option	1	1.0599	1.0599
UTI Banking Sector Fund - Growth	1.0502	1	1.0502
UTI Pharma and Healthcare Fund - Growth	1	1	1.1681
UTI Services Industries Fund-Growth Option	0.9574	0.9712	0.9764

Source: Calculated.

Table A8 : Order- α Efficiency Score (α=1)

Fund Name	*Output Oriented Approach*	*Input Oriented Approach*	*Graph Hyperbolic Approach*
Canara Robeco FORCE Fund - Institutional - Growth	1	1	1
Canara Robeco FORCE Fund - Retail - Growth	1	1	1
Franklin Infotech Fund - Growth	1	1	1
Franklin Pharma Fund - Growth	1	1	1
ICICI Prudential Banking and Financial Services Fund - Retail - Growth	1	1	1
ICICI Prudential Technology Fund - Growth	1	1	1
Reliance Banking Fund - Growth	1	1	1
Reliance Pharma Fund-Growth Plan-Growth	1	1	1
Religare Banking Fund - Regular - Growth	1	1	1
Sahara Banking and Financial Services Fund - Growth	1	1	1
Tata Life Sciences & Technology Fund - Growth	1	1	1
Sundaram Financial Services Opportunities Fund - Retails - Growth	1	1	1
UTI - MNC Fund (UGS 10000)-Growth Option	1	1	1
UTI Banking Sector Fund - Growth	1	1	1
UTI Pharma and Healthcare Fund - Growth	1	1	1
UTI Services Industries Fund-Growth Option	0.9574	0.9712	0.9712

Source: Calculated.