A Single Period Stochastic Model for Maximising Firm's Value

J. P. Singh*

Abstract

This article sets up a single period value maximisation model for the firm based on stochastic end-of-period cash inflows, stochastic bankruptcy costs and taxes based on income rather than wealth. The risk- return tradeoff is captured in the Capital Asset Pricing Model.Thus, the model also assumes a perfect capital market and market equilibrium. The model establishes the existence of a unique optimal financial leverage at which the firm value is maximised, this leverage being less than the maximum debt capacity of the firm.

Keywords: Firm Value, Debt Capacity, Capital Structure, Financial Leverage, Capital Markets

JEL Classification: G34, M41

Introduction

The policies and procedures of corporate management have undergone a complete metamorphosis in the preceding three decades. As the market efficiencies have approached perfection and trade boundaries have been belittled by technology, the world has dwindled to a shopping plaza. Competition has become unprecedentedly intense. Amidst all this, opportunities for investment have increased manifold. Corporates are taking recourse to innovative business strategies to facilitate cost reductions, competitiveness, and sustenance. The race is to deliver value to the investor. Thus, enhancing firm value has become a cardinal strategic objective for all market participants.

The association between financial leverage and firm value has been much explored. The traditional viewpoint believes that the value of a firm is a concave function of the proportion of debt employed in its capital structure, so that there exists a unique optimal level of financial leverage corresponding to which the firm's value attains a maximum. This position was radically challenged by Miller and Modigliani (MM hereinafter) in their celebrated leverage irrelevance propositions wherein they established through arbitrage arguments that the value of a firm was independent of its capital structure. However, MM made certain drastic assumptions in arriving at their conclusions viz. (i) substitutability of corporate debt by personal debt; (ii) absence of differentials between corporate and personal taxes (Modigliani & Miller, 1958, 1963, 1969). In the absence of empirical support, Miller later, conceded that although the leverage irrelevance did hold in steady state equilibrium, it was during the transient dynamic adjustment towards this point of equilibrium that a firm may achieve a value maximum (Miller, 1977). It was later established, while allowing for more realistic assumptions including bankruptcy and agency costs, non-debt corporate tax shields, depreciation and investment tax credits in addition to the conventional debt related tax offsets that a unique leverage for firm maximum did exist (DeAngelo & Masulis, 1980). A stochastic dynamic programming model for the determination of the optimal capital structure with a state preference setup for capturing the risk-return tradeoff, and the existence of bankruptcy costs has been attempted (Kraus & Litzenberger, 1973). One can also conclude the existence of an optimal leverage for the firm on the basis of market imperfections (Scott, 1976). Numerous attempts to optimise the capital structure based on varying premises, other than the above, have been reported as well (Scott, 1972; Myers, 1977; Brennan & Schwartz, 1978: Holland & Myers, 1980: Myers & Majluf, 1984; Hochman & Palmon, 1985). The relationships between the various determinants of firm value have also been thoroughly explored in

* Professor, Department of Management Studies, IIT Roorkee, Uttarakhand, India. Email: jpsiitr@gmail.com

several empirical studies e.g. between the investment and financing decisions of the firm (Dotan & Ravid, 1985; Dammon & Senbet, 1988; Green & Talmor, 1985) and between investment and debt tax shields (Kim & Wu, 1988; Mandelker & Rhee, 1984; Bradley, Jarrell, & Kim, 1984; Boquist & Moore, 1984; Titman & Wessels, 1988). Impact of inflation on the firm's leverage has also been empirically tested establishing that firms borrow more in times of rising inflation due to a fall in the real cost of debt (Jaffe, 1978; Modigliani & Cohn, 1979: Modigliani, 1982; Gordon & Malkiel, 1981; Gordon, 1982). The association between earnings volatility and leverage has been examined in reported literature showing that the optimal leverage tends to fall as the earnings volatility rises (Bradley *et al*., 1984; Titman & Wessels, 1988).

In this paper, we study a single period model for the maximisation of the value of the firm as a function of its leverage with stochastic terminal value and stochastic bankruptcy costs. We also assume the existence of income taxes and capture the risk-return tradeoff by the CAPM (Sharpe, 1964) thereby also postulating the existence of a perfect capital market. The assumptions are systematically presented below:

Assumptions of the Model

(a) The financing decision of the firm is segregated from the investment decision i.e. we assume that the investments of the firm have been identified and crystallised. However, the financing plan thereof is yet to be formulated. In fact, it is the objective of this article to set up a methodology for deriving a financing plan that would lead to a maximisation of the firm's wealth under the given assumptions.

(b) This is a single period model. Thus, the firm makes the investment at $t = 0$ in acquiring the productive assets and liquidates all the assets at $t = 1$ that remain after meeting the obligations of all non-capital factors of production. The terminal value of the firm is assumed to be stochastic.

(c) The income tax structure assumed here is one in which interest payments are tax-deductible and hence, result in interest tax shields whereas principal repayments are not so deductible. This tax system is the prevalent mode of corporate taxation in most countries.

(d) Bankruptcy costs are taken as a function of the terminal value (which is assumed to be a stochastic variable) of the firm. The exact nature of such costs depends on the type of bankruptcy (Ang, Chua, & McConnell, 1982; Warner, 1977). In the event of liquidation, these costs would include the losses resulting from the sale/disposal/liquidation of the physical assets of the firm at "distress prices" which are usually well below the fair/economic value of the respective assets. On the other hand, if a reorganisation is carried out, bankruptcy costs would include implicit costs like potential loss of sales due to reliability erosion in the customers' perceptions, greater cost and difficulty in obtaining credit, loss of goodwill in the employee market and consequent higher compensations demanded by quality manpower, cost escalations emanating from massive delays in the implementation of liquidation/reorganisation schemes etc. Costs of bankruptcy also include the costs associated with obtaining the decrees and implementing the settlements including professional fees and other expenses of lawyers, accountants, trustees, valuers etc. and other administrative expenses.

(e) We assume that the risk-return tradeoff is adequately captured by the Capital Asset Pricing Model (CAPM hereinafter) so that the model pre-supposes the existence of a perfect capital market with equilibrium dynamics.

Setting up the Model

The Unlevered Firm

As is the practice, we consider an unlevered (all equity financed) firm to start with. The firm invests a sum of I units of money at $t = 0$. This investment yields a terminal value of $\tilde{T}$ at $t=1$ after meeting the claims of all non-capital factors of production (the tilde ~ shall throughout this article indicate the stochastic nature of the underlying variable). Since this is a single-period model and all assets are liquidated at $t=1$, the gross earnings of the firm for this period are $\tilde{T} - I$. Since the firm is unlevered, there is no debt and hence no interest, so that the post-tax cashflows are simply $\tilde{T} - \tau\left(\tilde{T} - I\right)$ where τ is the tax rate. The return on equity for the unlevered firm $\tilde{R}_u$ is, therefore,

$$\tilde{R}_u = \frac{\tilde{T} - \tau\left(\tilde{T} - I\right)}{E_u} - 1 \tag{1}$$

Where E_u is the market value of the firm's equity (that equal V_u, the market value of the firm, since the firm is unlevered). We also have

$$E\left(\tilde{R}_u\right) = \frac{(1-\tau)E\left(\tilde{T}\right) + \tau I}{E_u} - 1 \quad ;$$

$$Cov\left(\tilde{R}_u, \tilde{R}_m\right) = \frac{(1-\tau)}{E_u} Cov\left(\tilde{T}, \tilde{R}_m\right) \tag{2}$$

Hence, we have, by the CAPM,

$$(1-\tau)E\left(\tilde{T}\right) + \tau I - E_u = E_u R_f + (1-\tau)\beta_{\tilde{T}}\left(\tilde{R}_m - \tilde{R}_f\right) \tag{3}$$

The Levered Firm

Using the notation as above, to the extent applicable,we, now, assume that part of the investment is financed by debt (D), the interest rate, R_d whereof, is a function of the amount of debt. In this case, the above analysis needs to be modified on several counts viz.

(a) the market value of the levered firm is the aggregate of the market value of the levered firm's equity and its debt i.e. $V_l = E_l + D$;

(b) if the firm's resources are adequate to meet all the claims of the lendersviz. interest payments and principal repayments, the taxable earnings are $\tilde{T} - I - R_d D$ and the post-tax cash flows before meeting debt obligations are $\tilde{T} - \tau\left(\tilde{T} - I - R_d D\right)$ with the corresponding levered return on equity $\tilde{R}_l$ being

$$\tilde{R}_l = \frac{\tilde{T} - \tau\left(\tilde{T} - I - R_d D\right) - (1 + R_d)D}{E_l} - 1$$

$$= \frac{(1-\tau)\ \ \tilde{T} - (1 + R_d)D\ \ + \tau(I - D)}{E_l} - 1 \tag{4}$$

In this case, we have, $Cov\left(\tilde{R}_l, \tilde{R}_m\right) = \dfrac{(1-\tau)}{E_l} Cov\left(\tilde{T}, \tilde{R}_m\right)$

and, by the CAPM,

$$(1-\tau)E\left(\tilde{T}\right) + \tau(I - D) - (1-\tau)(1 + R_d)$$

$$D - E_l = E_l R_f + (1-\tau)\beta_{\tilde{T}}\left(\tilde{R}_m - R_f\right) \tag{5}$$

It is easily seen that eqs. (4) & (5) reduce to eqs. (2) & (3) respectively for *D=0*.

(c) if the claims of the lenders exceed the firm's terminal value $\tilde{T}$ i.e. $\tilde{T} < (1 + R_d)D$, bankruptcy occurs. The entire terminal value gets attached to the lenders and the return on equity falls to *-1* indicating full loss of capital invested, but not below *-1*in view of the "limited liability" of common stockholders. The return on debt falls from R_d to $\bar{R}_d = \left(\tilde{T} - \tilde{B} - D\right)/D$ since part of the terminal value shall be consumed in meeting the bankruptcy costs $\tilde{B}$ and also for the repayment obligations. To keep the exposition simple and tractable, we model the bankruptcy costs as a linear function of the (stochastic) terminal value i.e. $\tilde{B} = a + b\tilde{T}$ ·

A unified description of all the scenarios can be presented by introducing a bankruptcy operator $\tilde{\varepsilon}$ defined as:

$$\tilde{\varepsilon} = \begin{array}{l} 0\ if\ \ bankruptcy\ does\ not\ occur\ i.e.\ \tilde{T}\quad (1 + R_d)D \\ 1\ if\ \ bankruptcy\ does\ \ occur\ i.e.\ \tilde{T} < (1 + R_d)D \end{array} \tag{6}$$

and the generalised return on debt

$$\tilde{R}_d = (1 - \tilde{\varepsilon})R_d + \tilde{\varepsilon}\bar{R}_d \tag{7}$$

We, then, have

$$\tilde{R}_l = \frac{(1-\tau)\ \ \tilde{T} - \left(1 + \tilde{R}_d\right)D - \tilde{\varepsilon}\tilde{B}\ \ + \tau(1 - \tilde{\varepsilon})(I - D)}{E_l} - 1 \tag{8}$$

whence,

$$üüüü(üüüü,üüüüü_m)üüü = \frac{(1-\tau)}{E_l} \quad (\tilde{\ }, \tilde{\ }_m) - \quad (\tilde{\ }_d, \tilde{\ }_m)$$

$$- Cov\left(\tilde{\varepsilon}\tilde{B}, \tilde{R}_m\right)\ - \frac{\tau(I - D)}{E_l} Cov\left(\tilde{\varepsilon}, \tilde{R}_m\right)$$

$$= \frac{(1-\tau)}{E_l}\ Cov\left(\tilde{T}, \tilde{R}_m\right) - Cov\left(\tilde{\varepsilon}\tilde{B}, \tilde{R}_m\right)\ - \frac{\tau(I - D)}{E_l} Cov\left(\tilde{\varepsilon}, \tilde{R}_m\right)$$

(9)

on the assumption that debt returns and market returns are uncorrelated, whence $Cov\left(\tilde{R}_d, \tilde{R}_m\right) = 0$ and $\beta_{\tilde{R}_d} = 0$ giving $E\left(\tilde{R}_d\right) = R_f$. The CAPM, then, yields

$$(1-\tau)E\left(\tilde{T}\right) - (1-\tau)D - (1-\tau)DR_f - (1-\tau)E\left(\tilde{\varepsilon}\tilde{B}\right) + \tau(I-D)$$

$$-\tau(I-D)E(\tilde{\varepsilon}) - E_l = E_l R_f + \left[(1-\tau)\left(\beta_{\tilde{T}} - \beta_{\tilde{\varepsilon}\tilde{B}}\right)\right.$$

$$\left. -\tau(I-D)\beta_{\tilde{\varepsilon}}\right]\left(\tilde{R}_m - R_f\right)$$

or

$$(1-\tau)E\left(\tilde{T}\right) = \left[E_l + (1-\tau)D\right]\left(1+R_f\right) - \tau(I-D)$$

$$+\tau(I-D)\left[E(\tilde{\varepsilon}) - \beta_{\tilde{\varepsilon}}\left(\tilde{R}_m - R_f\right)\right] + (1-\tau)\beta_{\tilde{T}}\left(\tilde{R}_m - R_f\right)$$

$$+(1-\tau)\left[E\left(\tilde{\varepsilon}\tilde{B}\right) - \beta_{\tilde{\varepsilon}\tilde{B}}\left(\tilde{R}_m - R_f\right)\right] \tag{10}$$

Eliminating $(1-\tau)E\left(\tilde{T}\right)$ between eqs. (3) and (10), we get

$$E_u\left(1+R_f\right) = \left[E_l + (1-\tau)D\right]\left(1+R_f\right) + \tau D + \tau(I-D)$$

$$\left[E(\tilde{\varepsilon}) - \beta_{\tilde{\varepsilon}}\left(\tilde{R}_m - R_f\right)\right] + (1-\tau)\left[E\left(\tilde{\varepsilon}\tilde{B}\right) - \beta_{\tilde{\varepsilon}\tilde{B}}\left(\tilde{R}_m - R_f\right)\right] \tag{11}$$

Noting that $E_u = V_u$ and $E_l + D = V_l$, we get

$$V_u = V_l - \frac{R_f}{1+R_f}\tau D + \frac{\tau(I-D)}{1+R_f}\left[E(\tilde{\varepsilon}) - \beta_{\tilde{\varepsilon}}\left(\tilde{R}_m - R_f\right)\right]$$

$$+\frac{(1-\tau)}{1+R_f}\left[E\left(\tilde{\varepsilon}\tilde{B}\right) - \beta_{\tilde{\varepsilon}\tilde{B}}\left(\tilde{R}_m - R_f\right)\right] \tag{12}$$

Maximising the Firm's Value

The objective of this analysis is to maximise the value of the levered firm (V_l) by employing the optimal financial leverage for a given investment. However, an amount of borrowing D from the market at $t=0$ corresponds to a total payment obligation of $\rho \equiv \left(1+R_d\right)D$ at the end of the single period of the model ($t=1$). In other words, a firm must assure the availability of an amount $\rho \equiv \left(1+R_d\right)D$ at $t=1$ to the lenders to precipitate a borrowing of D at $t=0$. Incremental commitments by the firm shall lead to additional borrowings. This will be the situation so long as the firm does not attain its maximum borrowing capacity. Thereafter, incremental commitments shall not contribute to additional borrowings.

For the moment, we assume that the firm has not reached its maximum borrowing capacity. A contrary assumption would make the entire analysis redundant. If the maximum possible borrowing has already been achieved but the optimal leverage has not been reached, then, it follows that the optimal leverage can never be reached because the firm cannot borrow any more beyond its maximum borrowing capacity.

Differentiating eq. (10) with respect to $\rho \equiv \left(1+R_d\right)D$ (because the degree of freedom is not the amount of borrowing D but rather the amount that the firm can commit out of its resources at $t=1$ to service the borrowings), we obtain

$$\frac{dV_l}{d\rho} = \frac{R_f}{1+R_f}\tau\frac{dD}{d\rho} - \tau(I-D)\frac{d\chi_{\tilde{\varepsilon}}}{d\rho} + \tau\chi_{\tilde{\varepsilon}}\frac{dD}{d\rho} - (1-\tau)\frac{d\chi_{\tilde{\varepsilon}\tilde{B}}}{d\rho} \tag{13}$$

where $\chi_{\tilde{\varepsilon}} = \dfrac{E(\tilde{\varepsilon}) - \beta_{\tilde{\varepsilon}}\left(\tilde{R}_m - R_f\right)}{1+R_f}$ and

$\chi_{\tilde{\varepsilon}\tilde{B}} = \dfrac{E\left(\tilde{\varepsilon}\tilde{B}\right) - \beta_{\tilde{\varepsilon}\tilde{B}}\left(\tilde{R}_m - R_f\right)}{1+R_f}$ are both non-negative terms that are associated with the event of bankruptcy of the firm. Setting $\dfrac{dV_l}{d\rho} = 0$, we obtain,

$$\frac{dD}{d\rho} = \frac{\tau(I-D)\dfrac{d\chi_{\tilde{\varepsilon}}}{d\rho} + (1-\tau)\dfrac{d\chi_{\tilde{\varepsilon}\tilde{B}}}{d\rho}}{\tau\left[\dfrac{R_f}{1+R_f} + \chi_{\tilde{\varepsilon}}\right]} \tag{14}$$

provided that either factor in the denominator does not vanish. To see that eq. (14) represents a maximum value of V_l, we note the following:

(a) When $\rho \to \tilde{T}_{\min}$, the chances of bankruptcy are virtually nonexistent so that (i) $\dfrac{dD}{d\rho} \equiv \dfrac{dD}{dD\left(1+R_d\right)} \to \dfrac{1}{1+R_f}$, since default risk is absent and so $R_d \to R_f$; (ii)

$\chi_{\tilde{\varepsilon}} \to 0$; (iii) $\frac{d\chi_{\tilde{\varepsilon}}}{d\rho} \to 0$ and (iv) $\frac{d\chi_{\tilde{B}\tilde{\varepsilon}}}{d\rho} \to 0$ .In this case, $\frac{dV_l}{d\rho} > 0$ so that the value of the levered firm increases with an increase in the commitments to lenders.

(b) However, when $\rho = (1+R_d)D_{\max}$, where D_{max} is the maximum borrowing capacity of the firm, then, (i) $\frac{dD}{d\rho} \to 0$ because the firm cannot borrow any more from the market by committing more funds to service such debt; the chances of bankruptcy are significant so that (ii) $\frac{d\chi_{\tilde{\varepsilon}}}{d\rho} > 0$ and (iii) $\frac{d\chi_{\tilde{B}\tilde{\varepsilon}}}{d\rho} > 0$ as an increase in the amount of borrowings shall result in an increase in the chances of bankruptcy. In this case, $\frac{dV_l}{d\rho} < 0$ so that the value of the levered firm decreases with an increase in the commitments to service lenders.

(c) In view of (a) & (b) above, $\frac{dV_l}{d\rho} > 0$ as $\rho \to \tilde{T}_{\min}$ and $\frac{dV_l}{d\rho} < 0$ at $\rho_{\max} = (1+R_d)D_{\max}$ and the assumed continuity of the functions involved, there must lie a point, say, $\rho^* = (1+R_d)D^*$ between these two values of ρ at which $\frac{dV_l}{d\rho} = 0$. Further, since the slope $\frac{dV_l}{d\rho}$ is decreasing as ρ increases, the point $\rho^* = (1+R_d)D^*$ is a maxima of V_l.

There is another important implication of the above analysis. $\frac{dV_l}{d\rho}$ is strictly negative at $\rho_{\max} = (1+R_d)D_{\max}$ and is decreasing from the left. Hence, the point $\rho^* = (1+R_d)D^*$ (at which $\frac{dV_l}{d\rho} = 0$) must lie before $\rho_{\max}$ i.e. $D^* < D_{\max}$. Thus, the optimal borrowing (that maximises the firm's market value) is less than the maximum amount that the firm is capable of borrowing i.e. the firm's debt capacity.

Computation of $\chi_{\tilde{\varepsilon}}$ & $\chi_{\tilde{\varepsilon}\tilde{B}}$

We, now, derive explicit expressions for the terms related to bankruptcy. We assume, for the purpose, that $\tilde{T}$ and $\tilde{R}_m$ are normally distributed.

Computation of $\chi_{\tilde{\varepsilon}}$

We have, the partial mean of $\tilde{T}$, as

$$\int_{-\infty}^{\rho} \tilde{T} f(\tilde{T}) d\tilde{T} = E(\tilde{T}) F(\rho) - \sigma^2 f(\rho) \quad (15)$$

where $\rho = (1+R_d)D$ and $F(\rho) = \int_{-\infty}^{\rho} f(\tilde{T}) d\tilde{T}$ is the probability that the firm will be bankrupt at the end of the period with a borrowing D and $f(\rho)$ is the probability density of ρ. We also have

$$Cov(\tilde{\varepsilon}, \tilde{R}_m) = E(\tilde{\varepsilon}\tilde{R}_m) - E(\tilde{\varepsilon}) E(\tilde{R}_m)$$
$$= \int_{-\infty}^{\rho} f(\tilde{T}) \left[\int_{-\infty}^{\infty} \tilde{R}_m g(\tilde{R}_m | T) d\tilde{R}_m - E(\tilde{R}_m) \right] d\tilde{T}$$
$$= \int_{-\infty}^{\rho} f(\tilde{T}) \left[E(\tilde{R}_m) + Cov(\tilde{T}, \tilde{R}_m) \frac{[\tilde{T} - E(\tilde{T})]}{\sigma^2} - E(\tilde{R}_m) \right] d\tilde{T}$$
$$= \frac{Cov(\tilde{T}, \tilde{R}_m)}{\sigma^2} \left[\int_{-\infty}^{\rho} \tilde{T} f(\tilde{T}) d\tilde{T} - E(\tilde{T}) F(\rho) \right] \quad (16)$$

From eqs. (15) & (16), we get

$$Cov(\tilde{\varepsilon}, \tilde{R}_m) = -Cov(\tilde{T}, \tilde{R}_m) f(\rho) \quad (17)$$

whence, using the definition

$\chi_{\tilde{\varepsilon}} = \frac{E(\tilde{\varepsilon}) - \beta_{\tilde{\varepsilon}}(\tilde{R}_m - R_f)}{1+R_f}$ and the fact that

$E(\tilde{\varepsilon}) = \int_{-\infty}^{\infty} \tilde{\varepsilon} f(\tilde{T}) d\tilde{T} = \int_{-\infty}^{\rho} f(\tilde{T}) d\tilde{T} = F(\rho)$, we have

$$\chi_{\tilde{\varepsilon}} = \frac{F(\rho) + \beta_{\tilde{T}}(\tilde{R}_m - R_f) f(\rho)}{1+R_f} \quad (18)$$

Computation of $\chi_{\tilde{\varepsilon}\tilde{B}}$

By definition, $\chi_{\tilde{\varepsilon}\tilde{B}} = \frac{E(\tilde{\varepsilon}\tilde{B}) - \beta_{\tilde{\varepsilon}\tilde{B}}(\tilde{R}_m - R_f)}{1+R_f}$. Now,

$$E\left(\tilde{\varepsilon}\tilde{B}\right)=\int_{-\infty}^{\infty}\tilde{\varepsilon}\tilde{B}f\left(\tilde{T}\right)d\tilde{T}=\int_{-\infty}^{\rho}\tilde{B}f\left(\tilde{T}\right)d\tilde{T}$$
$$=\int_{-\infty}^{\rho}\left(a+b\tilde{T}\right)f\left(\tilde{T}\right)d\tilde{T}=aF(\rho)+ \quad (19)$$
$$b\left[E\left(\tilde{T}\right)F(\rho)-\sigma^2 f(\rho)\right]$$

Further,

$$Cov\left(\tilde{\varepsilon}\tilde{B},\tilde{R}_m\right)=Cov\left(\tilde{\varepsilon}\left(a+b\tilde{T}\right),\tilde{R}_m\right)=aCov\left(\tilde{\varepsilon},\tilde{R}_m\right)+bCov\left(\tilde{\varepsilon}\tilde{T},\tilde{R}_m\right)$$
$$=-aCov\left(\tilde{T},\tilde{R}_m\right)f(\rho)+b\left[E\left(\tilde{\varepsilon}\tilde{T}\tilde{R}_m\right)-E\left(\tilde{\varepsilon}\tilde{T}\right)E\left(\tilde{R}_m\right)\right]$$
$$=-aCov\left(\tilde{T},\tilde{R}_m\right)f(\rho)+b\int_{-\infty}^{\rho}\tilde{T}f\left(\tilde{T}\right)\begin{bmatrix}\int_{-\infty}^{\infty}\tilde{R}_m g\left(\tilde{R}_m|T\right)d\tilde{R}_m\\-E\left(\tilde{R}_m\right)\end{bmatrix}d\tilde{T}$$
$$=-aCov\left(\tilde{T},\tilde{R}_m\right)f(\rho)+b\frac{Cov\left(\tilde{T},\tilde{R}_m\right)}{\sigma^2}$$
$$\left[\int_{-\infty}^{\rho}\tilde{T}^2 f\left(\tilde{T}\right)d\tilde{T}-E\left(\tilde{T}\right)\int_{-\infty}^{\rho}\tilde{T}f\left(\tilde{T}\right)d\tilde{T}\right]$$
$$=-aCov\left(\tilde{T},\tilde{R}_m\right)f(\rho)+b\frac{Cov\left(\tilde{T},\tilde{R}_m\right)}{\sigma^2}\left\{\sigma^2\left[F(\rho)-\rho f(\rho)\right]\right\}$$
$$=-aCov\left(\tilde{T},\tilde{R}_m\right)f(\rho)+bCov\left(\tilde{T},\tilde{R}_m\right)\left[F(\rho)-\rho f(\rho)\right]$$

whence, we have

$$\chi_{\tilde{\varepsilon}\tilde{B}}=\frac{aF(\rho)+b\left[E\left(\tilde{T}\right)F(\rho)-\sigma^2 f(\rho)\right]+\beta_{\tilde{T}}\left[(a+b\rho)f(\rho)-bF(\rho)\right]\left(\tilde{R}_m-R_f\right)}{1+R_f} \quad (20)$$

In deriving the aforesaid results, the following standard identities that hold for normal distributions, have been used (Feller, 1968; Mood & Graybill, 1963):

$$\int_{-\infty}^{\rho}\tilde{T}f\left(\tilde{T}\right)d\tilde{T}=E\left(\tilde{T}\right)F(\rho)-\sigma^2 f(\rho) \quad (21)$$

$$\int_{-\infty}^{\infty}\tilde{R}_m g\left(\tilde{R}_m|T\right)d\tilde{R}_m=E\left(\tilde{R}_m\right)+Cov\left(\tilde{T},\tilde{R}_m\right)\frac{\left[\tilde{T}-E\left(\tilde{T}\right)\right]}{\sigma^2} \quad (22)$$

$$\int_{-\infty}^{\rho}\tilde{T}^2 f\left(\tilde{T}\right)d\tilde{T}-E\left(\tilde{T}\right)\int_{-\infty}^{\rho}\tilde{T}f\left(\tilde{T}\right)d\tilde{T}=\sigma^2\left[F(\rho)-\rho f(\rho)\right] \quad (23)$$

Conclusions

We have set up a single period value maximisation model for the firm based on stochastic end-of-period cash inflows, stochastic bankruptcy costs and taxes based on income rather than wealth. The risk- return tradeoff is captured in the Capital Asset Pricing Model. Thus, the model also assumes a perfect capital market and market equilibrium. The model establishes the existence of a unique optimal financial leverage at which the firm value is maximised, this leverage being less than the maximum debt capacity of the firm.

References

Ang,J.,Chua, J., & McConnell, J. (1982). The administrative costs of corporate bankruptcy: A note. *Journal of Finance*, 37, 219.

Boquist, J. A., & Moore, W. T. (1984). Inter-Industry Leverage Differences and the De Angelo-Masulis Tax Shield Hypothesis. *Financial Management*, Spring 1984, 5.

Bradley, M., Jarrell, G., & Kim, E. H. (1984). On the existence of an optimal capital structure: Theory and evidence. *Journal of Finance*, 39, 857.

Brennan, M., & Schwartz, E. (1978). Corporate income taxes, valuation, and the problem of optimal capital structure. *Journal of Business*, 51, 103.

DeAngelo, H., & Masulis, R. W. (1980). Optimal capital structure under corporate and personaltaxes. *Journal of Financial Economics*, 8, 3.

Dammon, R. M., & Senbet, L. W. (1988). The effect of taxes and depreciation on corporateinvestment and financial leverage. *Journal of Finance*, 43, 357.

Dotan, A. & Ravid, S. A. (1985). On the interaction of real and financial decisions of thefirm under uncertainty.*Journal of Finance*, 40, 501.

Feller, W. (1968). *An introduction to probability theory and its application*, 1. John Wiley and Sons Inc., New York.

Gordon, R. H., & Malkiel, B. G. (1981). Corporation Finance, in *How Taxes Affect EconomicBehavior*, H.J. Aaron and J.A. Pechman (eds), (Washington, DC: The Brookings Institution, 1981).

Gordon, M. J. (1982). Leverage and the value of a firm under a progressive personal incometax.*Journal of Banking and Finance*, 6, 483.

Green, R., & Talmor, E. (1985). The Structure and Incentive Effects of Corporate Tax Liability. *Journal of Finance*, 40, 1095.

Hochman, S. & Palmon, O. (1985). The impact of inflation on the aggregate debt-asset ratio, *Journal of Finance*, 40, 1115.

Holland, D. M., & Myers, S. C. (1980). Profitability and Capital Cost for Manufacturing Corporations and All Non financial Corporations. *American Economic Review*, 320.

Jaffe, J. F. (1978). A note on taxation and investment. *Journal of Finance*, 33, 1439.

Kim, M. K., & Wu, C. (1988). Effects of Inflation on Capital Structure. *Financial Review,* 23, 183.

Kraus, A., & Litzenberger, R. (1973). A state preference model of optimal financial leverage. *Journal of Finance,* 28, 911.

Mandelker, G., & Rhee, S. G. (1984). The impact of the degrees of operating and financial leverage on systematic risk of common stock. *Journal of Financial and Quantitative Analysis,* 19, 45.

Miller, M. H. (1977). Debt and taxes. *Journal of Finance,* 32, 261.

Modigliani, F. F., & Miller, M. H. (1958).The cost of capital, corporation finance, and the theory of investment. *American Economic Review*, 48(3), 261.

Modigliani, F. F., & Miller, M. H. (1963). Corporation income taxes and the cost of capital: A correction. *American Economic Review*, 53(3), 433.

Modigliani, F. F., & Miller, M. H. (1969). Reply to Heins and Sprenkle, *American Economic Review,* 59(4), 592.

Modigliani, F. F., & Cohn, R. (1979). Inflation, rationed valuation and the market. *Financial Analysis Journal*, 24.

Modigliani, F. F. (1982). Debt, dividend policy, taxes, inflation and market valuation. *Journal of Finance,* 37, 255.

Mood, A. M., & Graybill, F. A. (1963). *Introduction to the theory of statistics.* McGraw-Hill, New York.

Myers, S. (1977). Determinants of Corporate Borrowing. *Journal of Financial Economics,* 5, 147.

Myres, S., & Majluf, N. (1984). Corporate financing and investment decisions when firms have information investors do not have. *Journal of Financial Economics,* 13, 187.

Scott, D. (1972). Evidence on the Importance of Financial Structure. *Financial Management,* 1, 45.

Scott, J. H. (1976). A theory of optimal capital structure. *The Bell Journal of Economics*, 7(1), 33.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, 19(3), 425.

Titman, S., & Wessels, R. (1988). The Determinants of Capital StructureChoice, *Journal ofFinance,* 43, 1.

Warner, J. (1977). Bankruptcy costs: Some evidence. *Journal of Finance*, 32, 337.