Reducing the gap between two MADM models

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Abstract— Several methods have been proposed for solving Multi-Attribute Decision-Making problems (MADM). A major criticism of MADM is that different techniques may yield different results when applied to the same problem. In this paper, we investigate the performance of two well-known MADM models: 1. AHP, and 2. TOPSIS. Although, there is no exact way to know which model gives the right answer. But, AHP was selected as the basis to which to compare the other methods, because it extremely popular in practice. Then, by changing the separation measures in TOPSIS model from P=2 (Euclidean distance) to another values (P≠2; i.e. 1.1, 1.2, etc., based on Birnbaum, 1998, p. 185), result are investigated.

Keywords— MADM, AHP, TOPSIS, Separation measures, Euclidean distance

1. INTRODUCTION

Multi-attribute decision making (MADM) models are selector models that are used for evaluating, ranking and selecting the most appropriate alternative from among several alternatives (Alinezhad and Amini, 2011). Several methods have been proposed for solving multi-attribute decision making problem (MADM). A major criticism of MADM is that different techniques may yield different results when applied to the same problem (Zanakis et al., 1998). In this paper, we investigated the performance of two well-known MADM models: 1. AHP, and 2. TOPSIS.

The AHP is a decision support tool which can be used to solve complex decision problems. It uses a multi-level hierarchical structure of objectives, criteria, sub criteria, and alternatives. The pertinent data are derived by using a set of pair wise comparisons. These comparisons are used to obtain the weights of importance of the decision criteria, and the relative performance measures of the alternatives in terms of each individual decision criterion. If the comparisons are not perfectly consistent, then it provides a mechanism for improving consistency (Triantaphyllou and Man, 1995). Also, TOPSIS (technique for order preference by similarity to an ideal solution), proposed by Yoon and Hwang, is one of the widely used techniques in multi-attribute decision making (MADM). TOPSIS can rank a finite number of feasible alternatives in order of preference according to the features of each attribute of every alternative and select a suitable alternative that conforms to the decision maker's ideal. The basic concept of TOPSIS technique is that the selected alternative will have the shortest Euclidean distance from the ideal solution and the farthest Euclidean distance from the anti-ideal solution (Lin et al., 2008). The present study presents a comparative analysis of different **P** values (in separation measures step) adopted in TOPSIS by testing them against AHP, to reducing the gap between AHP and TOPSIS model results.

The paper is organized as follows. In section 2, the literature and section 3, the proposed approach is discussed. Numerical example is provided in section 4. The paper is concluded in section 5.

2. LITERATURE REVIEW

Behavioral scientists, economists, and decision theorists have proposed a variety of models/methods describing how a DM might arrive at a preference judgment when choosing among multiple attribute alternatives (Hwang and Yoon, 1981). Such that, different MADM techniques yield different results for the same problem (Zanakis et al., 1998), (Mianabadi and Afshar, 2008), and (Ustinovichius et al., 2007). Therefore, it is not clear which approach is better. Voogd (1982) demonstrated that, in some qualitative and quantitative models (9 and 23 models, respectively), generally, can be said that there is a minimum of 40% chance that a technique results in another ranking than any other technique. Zanakis et al., (1998), described the inconsistency in such results occurs because: (a) the techniques use weights differently in their calculations; (b) algorithms differ in their approach to selecting the best solution; (c) many algorithms attempt to scale the objectives, which affects the weights already chosen; and (d) some algorithms introduce additional parameters that affect which solution will be chosen. In addition, have performed comparisons on ELECTRE, TOPSIS, MEW (Multiplicative Exponential Weighting), SAW and four versions of AHP (original vs. geometric scale and right eigenvector vs. mean transformation solution) using simulated data. The result indicated that, TOPSIS behaves closer to AHP and differently from ELECTRE and MEW techniques. Hwang and Yoon (1981) introduced three ordering techniques to reach a unified solution by employing some MADM methods, simultaneously. The first technique (Statistic) ranks alternatives according to their mean rankings. The second (Borda method) is based on a majority rule binary relation. the last procedures (Copeland method) is a modification of the majority rule case taking into account "losses" as well as "win". Olson (2004), reviewed several applications of TOPSIS using different weighting schemes (i.e. equal weights, centroid weights, and weights obtained by regression). Ustinovichius et al., (2007), described when Borda, Copeland and the Average weight methods are applied to the analysis of alternatives, the uncertainty about the choice of the best option still remains.

Therefore, proposed a multi criteria iterative decision method. When practically was used, the suggested technique demonstrated quick convergence of the iterative process and unambiguous ranking of the compared alternatives. Gelderman and Rentz (2000) compared the three MADM methods: SAR (Simple Additive Ranking), SAW and Promethee, and the striking similarities are obtained. Sennaroglu and Sen (2012) and Pawar and Verma (2013) applied an integrated AHP and TOPSIS approach; so that, the weights of criteria were determined using the AHP, and the ranking process was carried out by TOPSIS.

For the first time in this paper, the notion of the reducing the gap between two MADM models (AHP and Topsis) with using different separation measures in TOPSIS techniques (from: $P=2$, to: $P\neq 2$ and $1 <= P <= 2$) is introduced.

3. PROPOSED APPROACH

We assume that the reader has previous knowledge about the AHP and Topsis models; so, they will not be further described here (for more details see: Saaty (2000) for AHP and Hwang and Yoon (1981) for Topsis).

- What is the rationale for selecting AHP result as the benchmark?
- And which separation measures can be used in Topsis, to reducing the gap between AHP and Topsis model results? Are some of questions we need to deal with in this section.

As it was stated earlier, the AHP result was used as benchmark. In original Topsis model (step 4 of Topsis model) we have: calculate the distance of alternatives from ideal and anti-ideal solutions. For this, we usually use the Euclidean norm as follow (Alinezhad and Amini, 2011):

$$d_i^+ = \{\sum_j (V_{ij} - V_j^+)^2\}^{1/2} \quad ; i=1, 2,\dots, m \qquad (1)$$

$$d_i^- = \{\sum_j (V_{ij} - V_j^-)^2\}^{1/2} \quad ; i=1, 2,\dots, m \qquad (2)$$

Wherein, d_i^+ is the distance of the i^{th} alternative from the ideal solution and d_i^- is that of anti-ideal solution. Also, according to Birnbaum (1998) general characteristics of the Minkowski distance has define as follow:

$$d_{jk} = [\sum_{r=1}^{R} | X_{jr} - X_{kr} |^P]^{1/P} \quad (P >= 1) \qquad (3)$$

Where; X_{jr} and X_{kr} are the r^{th} coordinates of point's j and k, respectively, in a K-dimensional spatial representation.

On the other side, Kruskals, and Groenen suggested that: P≠2 and 1<= P <= 2 in Eq. (3), respectively (based on Birnbaum, 1998, p. 185-186). Therefore, theoretically there is no reason P values to be restricted to P=1, 2. Since, with changing the separation measures in TOPSIS model from P=2, to another value (1, 1.1, 1.2… 1.9, 2), similarities in the behavior of this method with benchmark (AHP) are investigated.

4. NUMERICAL EXAMPLE

The following simple example (with 4 alternatives and 3 criteria), illustrates above concepts and shows how a unified results for two methods of MADM (AHP and TOPSIS) can be obtained.

According to Saaty (2000; p. 455) views:

"In the AHP one needs to be careful with criteria measured on the same absolute scale. Criteria measured in dollars are a common example of this. The priority of each criterion must be equal to the sum of the measurements of alternatives divided by the sum of the measurements of the alternatives with respect to all such criteria. Only then can one normalize the measurements of the alternatives, weight them by these priorities and add to obtain the relative weights of the alternatives with respect to all these criteria".

This is the problem we wish to address here (in the absence of other standards, this results was used as the benchmark). Clearly, this is an assumption which is made here in order to study different model results.

Table1 The original data matrix of 4 alternatives

Cri. / Alt.	C1	C2	C3
A1	1	9	7
A2	5	3	9
A3	5	7	7
A4	9	1	5
Total	20	20	28

As seen from below table, the relative weights present the priorities. Therefore, **this result (A3 (.279) > A1 (.250) ≈ A2 (.250) > A4 (.221)); was used as the benchmark**. In order to compare the original TOPSIS model results with benchmark, we used the same numerical example and following results are obtained (table 3, and Fig.1).

Table2 Result of AHP according to Saaty views

Criteria / Priorities	20/68 =.294	20/68 =.294	28/68 =.412	----
----	C1	C2	C3	Composite priorities
A1	.05	.45	.25	.250
A2	.25	.15	.32	.250
A3	.25	.35	.25	.279
A4	.45	.05	.18	.221

Table3 Original TOPSIS results

Alt.	Relative closeness To the ideal	Normalized relative closeness
A1	.493	.242
A2	.472	.232
A3	.598	.294
A4	.471	.232

Notes: the criteria weights are taken from table 2; as follow: (Wj= .294, .294, .412).

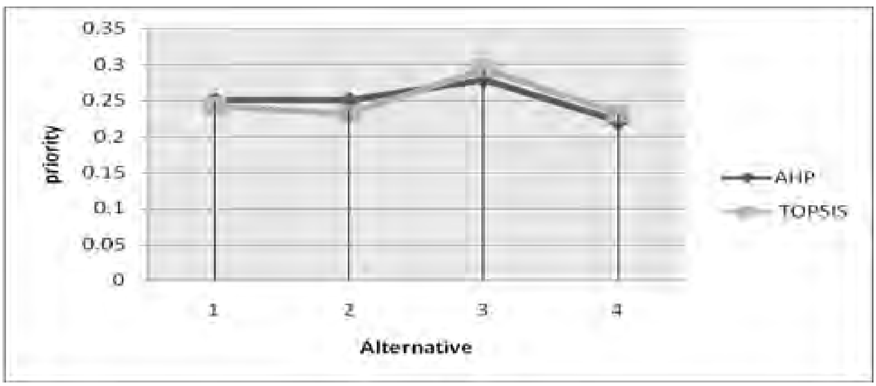

Fig.1 A comparison of results for same problem

As can be seen from table 3, the values in the two last columns are different from those of the two last column of table 2 (benchmark). Since, the different methods for deriving priorities in same problem can lead to different final choice.

Now, with changing of the separation measures in TOPSIS model, from: P=2, to: another values ((P≠2; and 1<= **P** <= 2), P=1, 1.1, 1.2… 1.9, 2), results are investigated. A comparison of test results is given in table 4 (i.e. Detail calculation for P=1.3 are presented in **Appendix I**).

Table4 TOPSIS model result with different P values

P=1			P=1.1			P=1.2			P=1.3		
RC To the ideal	NRC		RC To the ideal	NRC		RC To the ideal	NRC		RC To the ideal	NRC	
.494	.247		.494	.246		.494	.246		.494	.245	
.515	.258		.507	.253		.500	.249		.495	.245	
.596	.298		.596	.297		.596	.297		.597	.296	
.395	.197		.409	.204		.421	.209		.431	.214	
A3>A1≈A2>A4			A3>A2>A1>A4			A3>A2>A1>A4			A3>A2>A1>A4		

*RC=Relative Closeness

**NC=Normalized Relative Closeness

P=1.7			P=1.6			P=1.5			P=1.4		
RC To the ideal	NRC		RC To the ideal	NRC		RC To the ideal	NRC		RC To the ideal	NRC	
.494	.244		.494	.244		.494	.244		.493	.243	
.490	.243		.486	.240		.482	.238		.479	.236	
.597	.296		.597	.295		.597	.295		.598	.295	
.439	.217		.447	.221		.453	.224		.458	.226	
A3>A1>A2>A4			A3>A1>A2>A4			A3>A1>A2>A4			A3>A1>A2>A4		

P=2			P=1.9			P=1.8		
RC To the ideal	NRC		RC To the ideal	NRC		RC To the ideal	NRC	
.493	.243		.493	.243		.493	.242	
.477	.235		.475	.233		.472	.232	
.598	.294		.598	.294		.598	.294	
.463	.228		.467	.230		.471	.232	
A3>A1>A2≈A4			A3>A1>A2>A4			A3>A1>A2>A4		

Notes:

1. In the all of situations, the criteria weights are the same, as follows. (Wj= 0.294, 0.294, 0.412).
2. In the all of situations (except of P=2), separation measure was measured in absolute values term.

Finding:

1. As can be seen from table 4, the different P values (P≠2), can lead to different priorities (rank and preference intensities), in Topsis model.
2. Another important point to observe is that, the some of P value results are more effective than others. So that, only **P = 1.3** values is that the exhibit same ranking (with different, but closest preference intensities), with benchmark. A comparison of the test results is given in table 5, and fig. 2.
3. As can be seen from table, A3 ranking is best for the all of situations. And are the same as compared with benchmark.

Table5 A comparison results

---	priorities
benchmark	A1=.250 A2=.250 A3=.279 A4=.221
Topsis (with P=1.3)	A1=.245 A2=.245 A3=.296 A4=.214

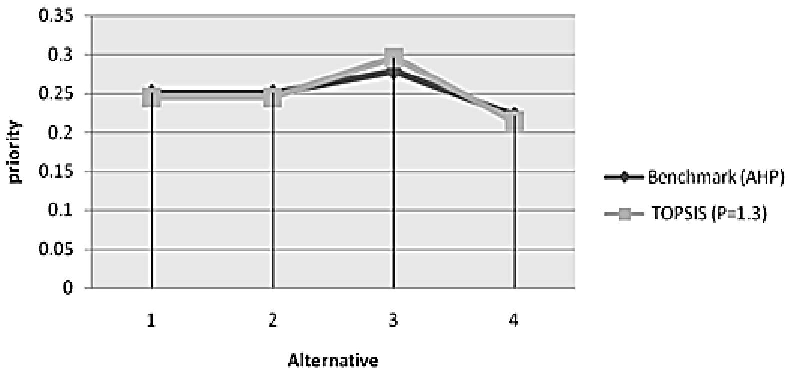

Fig.2 A comparison of results

As it was stated earlier, for further analysis, we generated 100 (i.e. 4*3, like Vargas example) <u>decision matrix with random absolute numbers</u>, in the range of 1 to 9 (in order to be consistent with the basic scale used by AHP). Then, similarities and differences in the behavior of these distances (p=1, 1.1, 1.2… 1.9, 2) are investigated. The priorities for the various **P** values were obtained and are shown here in table 6 and 7 (i.e. for one of decision matrix with random numbers, for complete decision matrix and results, see **Appendix II**).

Table 6 Decision matrix with random numbers

Alt.	C1	C2	C3	AHP results
A1	5	7	7	0.279
A2	7	9	1	0.250
A3	3	5	5	0.191
A4	1	9	9	0.279

Table 7 The various P values results (for table 6 data's)

P=1	0663*	0.565	0.325	0.676
	0.297**	0.253	0.146	0.303
1.1	0.664	0.549	0.335	0.663
	0.3	0.248	0.151	0.3
1.2	0.666	0.537	0.343	0.653
	0.303	0.244	0.156	0.297
1.3	0.667	0.526	0.35	0.643
	0.305	0.24	0.16	0.294
1.4	0.66	0.517	0.356	0.636
	0.307	0.237	0.164	0.292
1.5	0.67	0.509	0.362	0.629
	0.309	0.234	0.167	0.29
1.6	0.672	0.502	0.367	0.623
	0.311	0.232	0.17	0.288
1.7	0.673	0.496	0.372	0.618
	0.312	0.23	0.172	0.286
1.8	0.675	0.49	0.376	0.613
	0.313	0.228	0.175	0.285
1.9	0.676	0.485	0.38	0.609
	0.314	0.226	0.177	0.283
2	0.677	0.481	0.384	0.606
	0.315	0.224	0.179	0.282

*. Obtained Value

**. Normalized Value

Notice: Why absolute numbers used?

Here, due to comparison capability between AHP and TOPSIS model, absolute numbers is used. Clearly, this is an assumption that is made here in order to study different aggregation rules. Since, this could cause some bias in the result. Considering all above-mentioned points it that should be also considered. In summary, the main findings of this study are as follow (table 8, and fig. 3, 4 and 5).

1. If there are some equal alternative existed and one of them are not among elected alternative in top ranks, so these cases are not considered in internal calculations.

2. In all observation cases, if the city block distance (P=1), is utilized so, the distance of the positive ideal solution is became less and the distance of the nadir solution is became more.
3. Where, the distance from ideal solution is zero, the shortest distance to the ideal solution is guaranteed to have the longest distance to the negative ideal solution.

Table 8 The main finding in summary

No.	P value	accordance in %		Distance to ideal in % 2,3 (shortest distance to the ideal and longest distance to the negative solution)
		Top rank[1]	All rank	
1	1	84	51	**100***
2	1.1	**85***	50	92
3	1.2	84	**53***	90
4	1.3	84	50	89
5	1.4	84	51	89
6	1.5	84	51	86
7	1.6	84	49	83
8	1.7	84	51	82
9	1.8	84	50	81
10	1.9	84	50	80
11**	2	83	50	78

*. Maximum accordance

**. Original Topsis Values

Finding:

These results (table8, and fig. 3, 4, and 5) indicate that, the most accordance to <u>top rank</u> in 85% of cases, for P=1.1 and the least accordance in 83% of cases, for P=2 (the Euclidean distance); also, for <u>all ranks</u> the most accordance in 53% of cases, for P=1.2, and the least accordance in %49 of cases, for P= 1.6. Another interesting point is that, in accordance to Hwang & Yoon (1981, p. 136; for city block distance):

> *"Any alternative which has the shortest distance to the ideal solution is guaranteed to have the longest distance to the negative ideal solution (<u>this is not true for the Euclidean distance measure</u>)".*

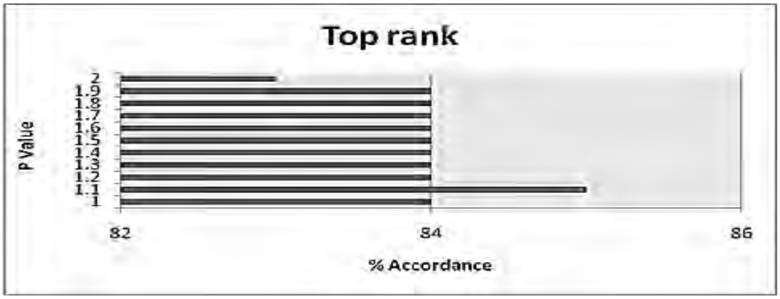

Fig. 3 Top rank accordance

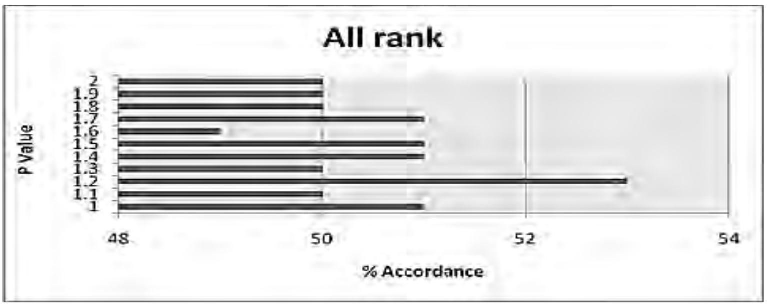

Fig. 4 All rank accordance

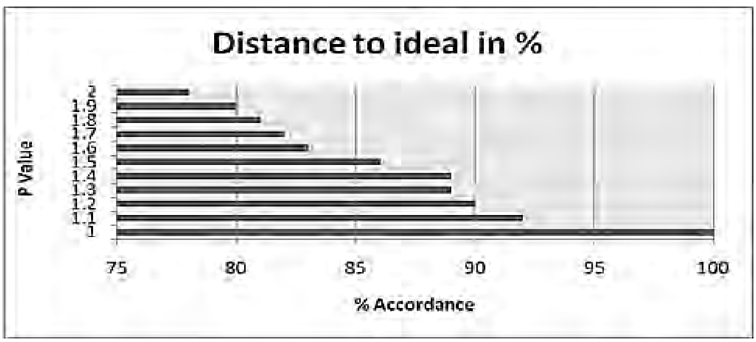

Fig. 5 Distance to ideal in %

(Shortest distance to the ideal and longest distance to the negative solution)

5. CONCLUDING REMARK

In summary, our paper intends to shed some light on the choice of the appropriate **P** values in separation measures step in Topsis model for reducing the gap between AHP and Topsis models result. The finding of this experimental study reveals that:

1. Some P values are better than others. So that, with changing of P values, TOPSIS behaves closer to AHP. Although, none of the values is completely accurate.
2. the comparative results of AHP (benchmark), and TOPSIS rank ordering reveals that, in average, 51% of cases for all P values (P=1, 51%; P=1.1, 50%; …; P=2, 50% - table 8), have the same ordering to the investigated alternatives. Therefore, we didn't find very significant differences in rank order with the changing of P values.
3. The earlier observations and conclusions (for distance to the ideal solution), by Hwang and Yoon (1981, p. 136 – (for P=1, p=2)), re-examined and same result re-observed.

In the future research, similar studies can be repeated for the use of the other MADM models with TOPSIS, and comparison results.

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APPENDIX I

Initial matrix	C1(+)	C2(+)	C3(+)
A1	1	9	7
A2	5	3	9
A3	5	7	7
A4	9	1	5

Normalized matrix	C1(+)	C2(+)	C3(+)
A1	0.087	0.761	0.490
A2	0.435	0.254	0.630
A3	0.435	0.592	0.490
A4	0.783	0.085	0.350

7	7	7
1	9	3
9	5	1
3	3	7

Weighted matrix	C1(+)	C2(+)	C3(+)
A1	0.026	0.224	0.202
A2	0.128	0.075	0.260
A3	0.128	0.174	0.202
A4	0.230	0.025	0.144

W_j	A+	A -
0.294	0.23	0.026
0.294	0.224	0.025
0.412	0.26	0.144

Relative Closeness	
c1*	0.495
c2*	0.495
c3*	0.598
c4*	0.431

Normalized Relative Closeness
0.245
0.245
0.296
0.214

Separation measures(+)	
d1+	0.234
d2+	0.215
d3+	0.165
d4+	0.271

Separation measures(-)	
d1-	0.229
d2-	0.211
d3-	0.245
d4-	0.205

i.e. for: **d1**+ = ((│ (.026-.230) │ 1..3 + (│ (.224-.224) │ 1..3 + (│ (.202-.260) │1..3)) (1/1..3) = 0.234

7. AUTHORS' PROFILE

Mohammad Azadfallah received his M.Sc. degree in industrial management from the Islamic Azad University, Science and Research Branch, Tehran. Iran in 2004. Presently he is working as a researcher at the business studies and development office, saipa yadak (after sales services organization for Iranian Automakers, Saipa).

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