A New Feature of Rank Reversal in Some of MADM Models

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Abstract

A remarkable result of Webber *et al*. (1996) shows the scale used, format and order in which the comparisons are entered in the matrix, may affect the AHP results. For the purpose of the experiment, in this investigation, we will concentrate our attention on format effect. In other words, we argue that, what happens if we change the format of alternative or attribute presentation in decision making matrix and redo our assessment of alternatives, in some of MADM models (i.e. TOPSIS, PROMETHEE II, ELECTRE, and three versions of AHP (distributive, ideal, and multiplicative modes)). The result showed that the stability of the TOPSIS, PROMETHEE II, and ELECTRE results. Meanwhile, the distributive, ideal, and multiplicative modes have bad and unreasonable performance. An interesting finding is that in distributive and ideal modes, for change the format of alternative presentation and in multiplicative mode, for change the format of attribute presentation, rank reversal phenomenon can be happened. The finding in this paper can help the decision maker select suitable prioritisation methods.

Keywords: MADM, TOPSIS, PROMETHEE II, ELECTRE, AHP, Format Effect, Rank Reversal

Introduction

Multi Attribute Decision Making (MADM) is the most well-known branch of decision-making. It is a branch of a general class of Operations Research (or OR) models which deal with decision problems under the presence of a number of decision criteria. This super class of models is very often called Multicriteria Decision Making (or MCDM). In the current literature, MCDM is divided into Multi Objective Decision Making (or MODM) and Multi Attribute Decision Making (or MADM). MODM studies decision problems in which the decision space is continuous. A typical example is mathematical programming problems with multiple objective functions. On the other hand, MADM concentrates on problems with discrete decision spaces. In these problems, the set of decision alternatives has been predetermined (Triantaphyllou *et al*., 1998). The latter method is preferred in this paper.

Multi Attribute Decision Making (MADM) models are selector models that are used for evaluating, ranking, and selecting the most appropriate alternative from among several alternatives (Alinezhad & Amini, 2011). Despite its usefulness, MADM models have some notable limitations, for example, rank reversal. So, rank order of the alternatives may change if we added or delete alternative(s) from the existing set of the alternatives. Alternative that is less preferred than another alternative becomes more preferred and vice versa (Endah & Raharjo, 2003). In the literature, many studies exist on rank reversal. However, most of them are limited to adding or deleting an alternative or criteria. But, Webber *et al*. (1996) have addressed that the format in which the comparisons are entered in the matrix, may affect the AHP results. Since, for the purpose of the experiment, in this paper we shall concentrate our attention on format effect in some of MADM models result (i.e. TOPSIS, PROMETHEE II, ELECTRE, and three versions of AHP (distributive, ideal, and multiplicative modes)).

The paper is organised as follow. The second section discusses rank reversal, a brief discussion of used MADM models, and format effect, while in third section literature

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is reviewed. Numerical example is provided in fourth section. The paper is concluded in fifth section.

Rank Reversal, MADM Models, and Format Effect

A description of rank reversal, MADM models, and format effect is presented as follows:

Rank Reversal

Rank reversal means that the ranking between two alternatives might be reversed after some variation occurs to the decision problem, like adding a new alternative, dropping an old alternative or replacing an old alternative by a worse one, etc. For example, two alternatives A_1 and A_2 may be initially ranked as $A_1 > A_2$ (i.e. A_1 is more preferable than A_2). After a new alternative A_3 is introduced into the decision problem and the alternatives are ranked again by using the same method, the ranking between A_1 and A_2 may be reversed and become $A_2 >$ A1 (Wang, 2007). There are two schools of thought about decision-making. One school maintains through an axiom that new alternatives that introduce no additional attributes should not cause rank reversal as that would violate rationality. The same procedure is used to rank alternatives in all situations. The other school maintains that the effect of other alternatives on the old rank depends on whether the decision maker's preference is influenced or not by the number and measurement of these alternatives. So, one procedure is used to preserve rank and another to allow it to change (Saaty, 2000). In other words, this phenomenon has been criticised from one side, and legitimised by another (Ishizaka & Labib, 2009). Which one of the two is right, can be an open-ended question.

MADM Models

In the present world, most of the problems given to the managers and even our everyday life problems have various dimensions, which are formulised with multiple variables. In other words, the final decision cannot be made with optimisation of just one variable. So, it is natural that solving of these problems becomes more complicated especially when the variables are in contradiction and increasing the desirability of one variable may require decreasing the desirability of other one. So Multi Criteria Decision Making (MCDM) and specially Multi Attribute Decision Making (MADM) have been developed for solving such kind of problems.

An MADM problem can be summarised in the form of a matrix in which the rows are different alternatives and the columns are the criteria specifying the properties of the alternatives. Also the matrix cells show the position of the row alternative in relation to the correspondent column criterion. Until how, we just presented the problem. Now to prioritize the alternatives, a decision technique is required that specifies an alternative of higher rank by interchanging and trading off of the different criteria (Amiri *et al*., 2008). In other words, the MADM models are an effective approach in dealing with this kind of decision problem.

We assume that the reader is familiar with the used MADM models in this paper (i.e. TOPSIS, PROMETHEE II, ELECTRE, AHP, and its variants) and do not reproduce the methodology here. So, a brief discussion is provided in this section.

TOPSIS

TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) proposed by Yoon and Hwang is one of the widely used techniques in Multi Attribute Decision making (MADM). TOPSIS can rank a finite number of feasible alternatives in order of preference according to the features of each attribute of every alternative and select a suitable alternative that conforms to the decision maker's ideal. The basic concept of TOPSIS technique is that selected alternative will have the shortest Euclidean distance from the ideal solution and the farthest Euclidean distance from the anti-ideal solution (Lin *et al*., 2008). For more details see Hwang and Yoon (1981).

PROMETHEE II

PROMETHEE (Preference Ranking Organisation Method for Enrichment Evaluations) methods were designed to treat Multi Criteria problems (Figueira *et al*., 2005). The evaluation table is the starting point of the PROMETHEE method. In this table, the alternatives are evaluated on the different criteria. These evaluations involve essentially numerical data. The implementation of PROMETHEE requires two additional types of information, namely:

- Information on the relative importance (i.e., the weights) of the criteria considered;
- Information on the decision maker's preference function, which he/she uses when comparing the contribution of the alternatives in terms of each separate criterion.

The PROMETHEE I partial ranking provides a ranking of alternatives. In some cases, this ranking may be incomplete. This means that some alternatives cannot be compared and therefore, cannot be include in a complete ranking. This occurs when the first alternative obtains high scores on particular criteria for which the second alternative obtains low scores and the opposite occurs for other criteria. The use of PROMETHEE I then suggests that the decision maker should engage in additional evaluation efforts. PROMETHEE II provides a complete ranking of the alternatives from the best to the worst one (Macharis *et al*., 2004). Here, the latter is our focus. For more details see Brans and Vincke (1985).

ELECTRE

Elimination Choice Translating Reality (ELECTRE) is a Multi Attribute Decision Making method for ranking multiple alternatives based on some criteria. This method is a very effective assessment solution that equips the decision activities with quantitative and qualitative features. In this method, the output is a set of ranks such that the necessary concordance will be provided in the most appropriate form. ELECTRE uses a new concept known as outranking. For example $A_k \rightarrow Al$ indicates that although k and l options do not have any priority to each other mathematically, the decision maker accept that A_k is better than A_l. In this method, all the alternatives are assessed using the outranking comparisons and the non-effective alternatives are omitted. Pair comparisons performed based on agreement rank of weights (W_j) and difference rank from weighting assessment values (V_{ij}) and are tested simultaneously for alternatives assessment. All these steps are planned according to a concordant and a discordant set that is known as concordance analysis (Amiri *et al*., 2008). For more details see Hwang and Yoon, (1981).

AHP

The Analytic Hierarchy Process (AHP) developed at the Wharton School of Business by Thomas Saaty, allows decision makers to model a complex problem in a hierarchical structure showing the relationships of the goal, objectives (criteria), sub-objectives, and alternatives. Uncertainties and other influencing factors can also be included.

AHP allows for the application of data, experience, insight, and intuition in a logical and thorough way. AHP enables decision makers to derive ratio scale priorities or weights as opposed to arbitrarily assigning them. In so doing, AHP not only supports decision makers by enabling them to structure complexity and exercise judgement, but allows them to incorporate both objective and subjective considerations in the decision process. AHP is a compensatory decision methodology because alternatives that are deficient with respect to one or more objectives can compensate by their performance with respect to other objectives. AHP is composed of several previously existing but unassociated concepts and techniques such as hierarchical structuring of complexity, pair wise comparisons, redundant judgments, an eigenvector method for deriving weights, and consistency considerations. Although each of these concepts and techniques was useful in and of themselves, Saaty's synergistic combination of the concepts and techniques (along with some new developments) produced a process whose power is indeed far more than the sum of its parts (Forman & Selly, 2001). For more details see Saaty (2000). In the distribute mode, the weights of the alternatives sum to one. It is used when there is dependence among the alternatives and a unit priority is distributed among them. The ideal mode is used to obtain the single best alternative regardless of what other alternatives there are. In the ideal mode, the local priorities of the alternatives under each criterion are divided by the largest value among them. This is done for each criterion, for each criterion one alternative becomes an ideal with value one (Figueira *et al*., 2005). For more details see Ishizaka and Labib (2009). Also, the main criticism of AHP has been the existence of rank reversal if alternatives are added or deleted (Wedley, 2001). Therefore, multiplicative AHP for rank preservation suggested. For more details see Wang (2007).

Format Effect

Format effect that means here, studying the effect of random (i.e. A_2, A_1, A_4, A_3, respectively) compared with non-random (i.e. A_1, A_2, A_3, A_4, respectively) presentation of comparisons, on the final score of alternatives.

Literature Review

Rank reversal phenomenon has become a topic of debate for many years (Endah & Raharjo, 2003). So, Tversky *et al*. (1990) explained three waves of studies of preference reversal. These studies may be briefly defined as follow. The first include the original experiments, the second wave of studies, conducted during the late 1970s and early 1980s, consisted of critical replications and attempts to eliminate preference reversals by procedural variations and by increased incentives, and the robustness of preference reversals led to the third wave of articles, which attempted to explain the phenomenon rather than eliminate it.

In the current literature, many studies exist on rank reversal phenomenon in some of MADM models. Here, we will mention some of them. For instance, for TOPSIS methodology: Ren *et al*., (2007), for PROMETHEE methodology: De Keyser and Peeters (1996), for ELECTRE methodology: Wang and Triantaphyllou (2006), and Wang (2007), for AHP (and its variants) methodology: Dyer (1990a, 1990b), Barzilai and Golany (1994), Triantaphyllou (2001), Ekart and Nemeth (2001), Finan and Hurley (2002), Endah and Raharjo (2003), Maleki and Hassanzadeh (2012), and for eight MADM models (ELECTRE, TOPSIS, Multiplicative Exponential Weighting (MEW), Simple Additive Weighting (SAW), and four version of AHP), Zanakis *et al*. (1998). In addition, Azadfallah (2014) discussed there are three factors that seemed to play a significant role in AHP rank reversals: 1. normalisation, 2. additive function, and 3. measurement scale. Nevertheless, many researchers tried to look at the effect of each of these factors individually; this paper sees all the factors together. However, most of them are limited to adding or deleting an alternative or criteria (attribute). As far as we know, only Webber *et al*. (1996) have addressed that the scale used, format and order in which the comparisons are entered in the matrix may affect the AHP results. Since, for the experiment in this paper, we will concentrate our attention on format effect. In other words, we argue that, what happens if we change the format of alternative or attribute presentation in decision making matrix and redo our assessment of alternatives, in some of the famous MADM models (i.e., TOPSIS, PROMETHEE II, ELECTRE, and three versions of AHP (distributive, ideal, and multiplicative modes)).

Numerical Example

The following simple example illustrates above concepts and shows how rank reversal phenomenon can be happened. Assume that, there are 6 alternatives (A_1, A_2… A_6) and 5 criteria (C_1, C_2… C_5). Some criteria have to be minimised (i.e., C_1 and C_5), the others have to be maximised (i.e., C_2, C_3, and C4). The performance values of the alternatives under these five criteria are shown in table 1.

Table 1: Performance Value

-	C_1^-	C_2	C_3	C_4	C_5^-
A_1	60	900	22	51	8
A_2	30	520	31	13	1
A_3	40	650	20	58	2
A_4	92	750	60	36	3
A_5	52	780	58	90	1
A_6	80	920	4	75	6

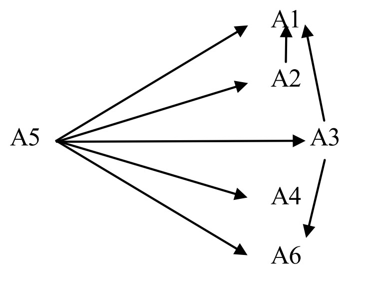
The following can be noticed from Table 1.

1. The equal weight rules were used. In accordance to Birnbaum (1998), several researchers have argued that the equal weight rule is often a highly accurate simplification of the decision making process. Thus, W_j= (0.2, 0.2, 0.2, 0.2, 0.2).
2. Caused by comparison capability between AHP and other models, absolute numbers is used. The following statement is evident.
    a. We have used absolute numbers rather than the relative numbers in decision matrix. This could cause some bias in the final result. But, in other side of coin, the comparison of the results (absolute number) with the ones of the AHP method (relative number, by: Webber *et al*., 1996) shows that, both methods (AHP with absolute and relative numbers) are extremely similar and rank reversal phenomenon can be happened.

As noted earlier, it is not the purpose of this paper to explain in detail the used MADM models. The result is shown in Table 2.

Table 2: MADM Model Results

Method	*Priority*
TOPSIS	$A_5 > A_3 > A_4 > A_2 > A_6 > A_1$ .261 .179 .172 .168 .116 .103
PROMETHEE II	$A_5 > A_2 > A_3 > A_6 > A_1 \approx A_4$ .56 0 -.04 -.12 -.20 -.20

Method	Priority
ELECTRE	
AHP distributive mode	$A_5 > A_2 > A_3 > A_4 > A_1 > A_6$ .246 .184 .160 .157 .130 .123
AHP ideal mode	$A_5 > A_2 > A_3 > A_4 > A_1 > A_6$ .878 .645 .587 .575 .507 .488
AHP * multiplicative mode	$A_5 > A_4 > A_2 > A_3 > A_1 > A_6$ .496 .314 .099 .048 .042 .002

*. For cost criteria: Xij^{-Wj}.

As can be seen in Table 2, often different MADM models yield different answers for the same problem. Further investigation on these methods would be interesting, but it is beyond the scope of this study.

Testing the Change of Attributes Format

A comparison of the result is given in Tables 3, 4, 5, 6, 7, and 8 (for TOPSIS, PROMETHEE II, ELECTRE, and three versions of AHP (distributive, ideal, and multiplicative modes, respectively)).

Table 3: TOPSIS Model Results (change of attributes format)

Format manipulated	Priority
1,5 *	$A_5 > A_3 > A_4 > A_2 > A_6 > A_1$ .261 .179 .172 .168 .116 .103
2,3 **	$A_5 > A_3 > A_4 > A_2 > A_6 > A_1$ .261 .179 .172 .168 .116 .103
2,4 **	$A_5 > A_3 > A_4 > A_2 > A_6 > A_1$ .261 .179 .172 .168 .116 .103
3,4 **	$A_5 > A_3 > A_4 > A_2 > A_6 > A_1$ .261 .179 .172 .168 .116 .103

*. Cost criteria

**. Benefit criteria (for all tables)

Table 4: PROMETHEE II Model Results (change of attributes format)

Format manipulated	Priority
1,5	$A_5 > A_2 > A_3 > A_6 > A_1 \approx A_4$ .56 0 -.04 -.12 -.20 -.20
2,3	$A_5 > A_2 > A_3 > A_6 > A_1 \approx A_4$ .56 0 -.04 -.12 -.20 -.20
2,4	$A_5 > A_2 > A_3 > A_6 > A_1 \approx A_4$ .56 0 -.04 -.12 -.20 -.20
3,4	$A_5 > A_2 > A_3 > A_6 > A_1 \approx A_4$ .56 0 -.04 -.12 -.20 -.20

Table 5: ELECTRE Model Results (change of attributes format)

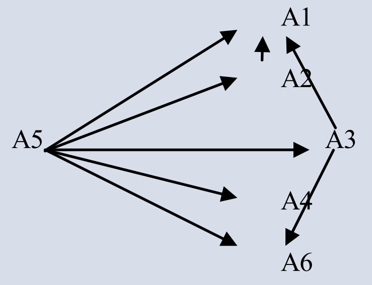
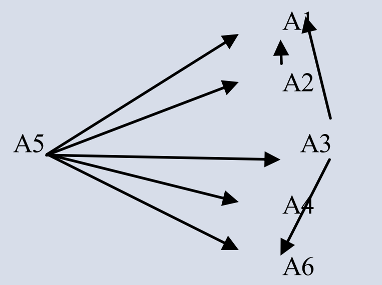
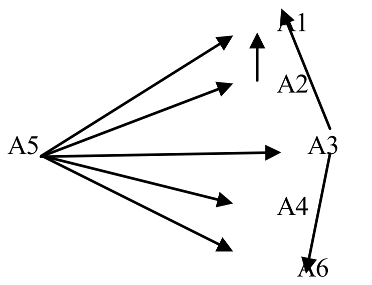
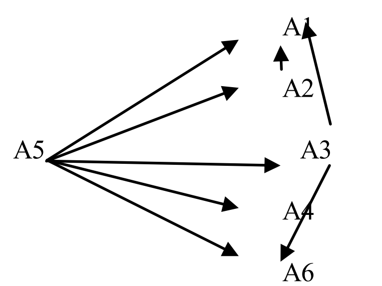
Format manipulated	Priority	Format manipulated	Priority
1,5		2,4	
2,3		3,4	

Table 6: AHP Model / Distribute Mode Results (change of attributes format)

Format manipulated	Priority
1,5	$A_5 > A_2 > A_3 > A_4 > A_1 > A_6$ .246 .184 .160 .157 .130 .123
2,3	$A_5 > A_2 > A_3 > A_4 > A_1 > A_6$ .246 .184 .160 .157 .130 .123
2,4	$A_5 > A_2 > A_3 > A_4 > A_1 > A_6$ .246 .184 .160 .157 .130 .123
3,4	$A_5 > A_2 > A_3 > A_4 > A_1 > A_6$ .246 .184 .160 .157 .130 .123

Table 7: AHP model / Ideal Mode Results (change of attributes format)

Format manipulated	Priority
1,5	$A_5 > A_2 > A_3 > A_4 > A_1 > A_6$ .878 .645 .587 .575 .507 .488
2,3	$A_5 > A_2 > A_3 > A_4 > A_1 > A_6$ .878 .645 .587 .575 .507 .488
2,4	$A_5 > A_2 > A_3 > A_4 > A_1 > A_6$ .878 .645 .587 .575 .507 .488
3,4	$A_5 > A_2 > A_3 > A_4 > A_1 > A_6$ .878 .645 .587 .575 .507 .488

Findings

Comparative results (from Tables 3-8, to table 2) indicated the stability of the TOPSIS, PROMETHEE, ELECTRE, distributive, and ideal modes of AHP results. In other words, the change in order of attribute presentation has no impact on the results (rank preservation).

Table 8: AHP Model / Multiplicative Mode Results (change of attributes format)

Format manipulated	Priority (normalised result)
1,5 *	$A_5 > A_4 > A_2 > A_3 > A_1 > A_6$ .496 .314 .099 .048 .042 .002
2,3	$A_5 > A_1 > A_4 > A_6 > A_3 > A_2$ .289 .182 .161 .146 .136 .086
2,4	$A_5 > A_4 > A_2 > A_3 > A_1 > A_6$ .496 .314 .099 .048 .042 .002
3,4	$A_5 > A_3 > A_6 > A_1 > A_4 > A_2$ .534 .158 .144 .093 .061 .010

*. For cost criteria: Xij^{-Wj}.

It is remarkable to observe that in this illustrative example, the multiplicative AHP has bad performance, and is the case of rank reversal phenomenon (table 8, for: 2 of 4 cases). In other words, the change in order of attribute presentation has a great impact on the results (rank reversal phenomenon).

Testing the Change of Alternatives Format

A comparison of the results is given in Tables 9, 10, 11, 12, 13, and 14 (for TOPSIS, PROMETHEE II, ELECTRE, and three versions of AHP (distributive, ideal, and multiplicative modes, respectively)).

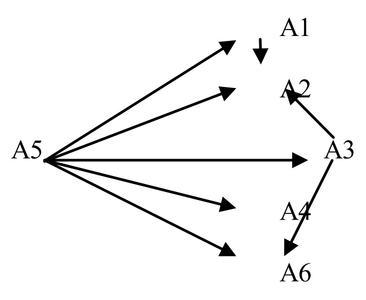
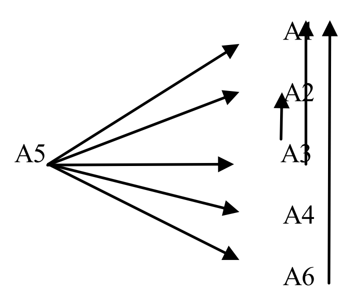
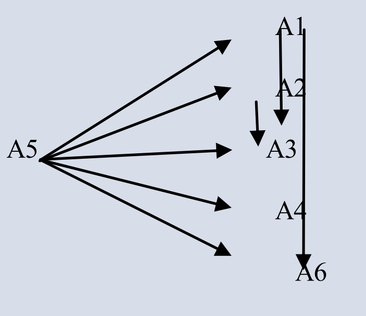
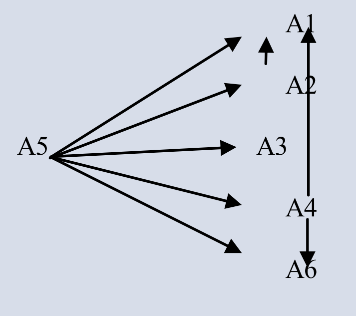
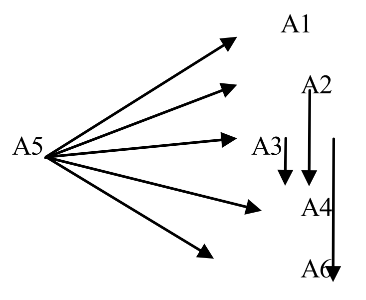
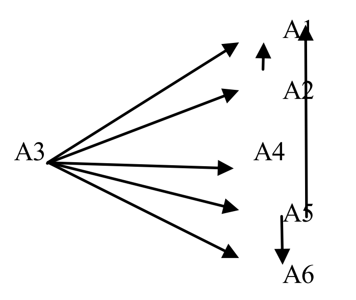
Table 9: TOPSIS Model Results (change of alternatives format)

Format manipulated	Priority	Format manipulated	Priority
1,2	$A_5 > A_3 > A_4 > A_1 > A_6 > A_2$ .261 .179 .172 .168 .116 .103	2,6	$A_5 > A_3 > A_4 > A_6 > A_2 > A_1$ .261 .179 .172 .168 .116 .103
1,3	$A_5 > A_1 > A_4 > A_2 > A_6 > A_3$ .261 .179 .172 .168 .116 .103	3,4	$A_5 > A_4 > A_3 > A_2 > A_6 > A_1$ .261 .179 .172 .168 .116 .103
1,4	$A_5 > A_3 > A_1 > A_2 > A_6 > A_4$ .261 .179 .172 .168 .116 .103	3,5	$A_3 > A_5 > A_4 > A_2 > A_6 > A_1$ .261 .179 .172 .168 .116 .103
1,5	$A_1 > A_3 > A_4 > A_2 > A_6 > A_5$ .261 .179 .172 .168 .116 .103	3,6	$A_5 > A_6 > A_4 > A_2 > A_3 > A_1$ .261 .179 .172 .168 .116 .103
1,6	$A_5 > A_3 > A_4 > A_2 > A_1 > A_6$ .261 .179 .172 .168 .116 .103	4,5	$A_4 > A_3 > A_5 > A_2 > A_6 > A_1$ .261 .179 .172 .168 .116 .103
2,3	$A_5 > A_2 > A_4 > A_3 > A_6 > A_1$ .261 .179 .172 .168 .116 .103	4,6	$A_5 > A_3 > A_6 > A_2 > A_4 > A_1$ .261 .179 .172 .168 .116 .103
2,4	$A_5 > A_3 > A_2 > A_4 > A_6 > A_1$ .261 .179 .172 .168 .116 .103	5,6	$A_6 > A_3 > A_4 > A_2 > A_5 > A_1$ .261 .179 .172 .168 .116 .103
2,5	$A_2 > A_3 > A_4 > A_5 > A_6 > A_1$ .261 .179 .172 .168 .116 .103	-	-

Table 10: PROMETHEE Model Results (change of alternatives format)

Format manipulated	*Priority*	*Format manipulated*	*Priority*
1,2	$A_5 > A_1 > A_3 > A_6 > A_2 \approx A_4$ .56 0 -.04 -.12 -.20 -.20	2,6	$A_5 > A_6 > A_3 > A_2 > A_1 \approx A_4$ .56 0 -.04 -.12 -.20 .20
1,3	$A_5 > A_2 > A_1 > A_6 > A_3 \approx A_4$ .56 0 -.04 -.12 -.20 -.20	3,4	$A_5 > A_2 > A_4 > A_6 > A_1 \approx A_3$ .56 0 -.04 -.12 -.20 -.20
1,4	$A_5 > A_2 > A_3 > A_6 > A_1 \approx A_4$ .56 0 -.04 -.12 -.20 -.20	3,5	$A_3 > A_2 > A_5 > A_6 > A_1 \approx A_4$ .56 0 -.04 -.12 -.20 -.20
1,5	$A_1 > A_2 > A_3 > A_6 > A_4 \approx A_5$ .56 0 -.04 -.12 -.20 -.20	3,6	$A_5 > A_2 > A_6 > A_3 > A_1 \approx A_4$ .56 0 -.04 -.12 -.20 -.20
1,6	$A_5 > A_2 > A_3 > A_1 > A_4 \approx A_6$ .56 0 -.04 -.12 -.20 -.20	4,5	$A_4 > A_2 > A_3 > A_6 > A_1 \approx A_5$ .56 0 -.04 -.12 -.20 -.20
2,3	$A_5 > A_3 > A_2 > A_6 > A_1 \approx A_4$ .56 0 -.04 -.12 -.20 -.20	4,6	$A_5 > A_2 > A_3 > A_4 > A_1 \approx A_6$ .56 0 -.04 -.12 -.20 -.20
2,4	$A_5 > A_4 > A_3 > A_6 > A_1 \approx A_2$ .56 0 -.04 -.12 -.20 -.20	5,6	$A_6 > A_2 > A_3 > A_5 > A_1 \approx A_4$ .56 0 -.04 -.12 -.20 -.20
2,5	$A_2 > A_5 > A_3 > A_6 > A_1 \approx A_4$ .56 0 -.04 -.12 -.20 -.20	-	-

Table 11: ELECTRE Model Results (change of alternatives format)

Format manipulated	*Priority*	*Format manipulated*	*Priority*
1,2		2,6	
1,3		3,4	
1,4		3,5	

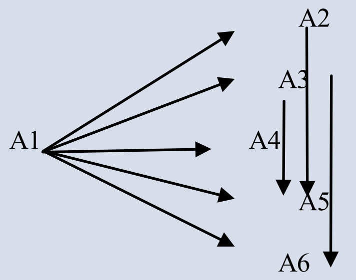
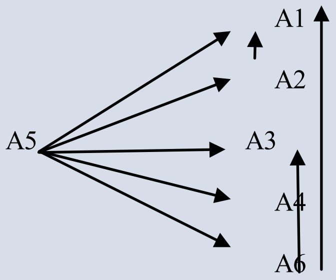
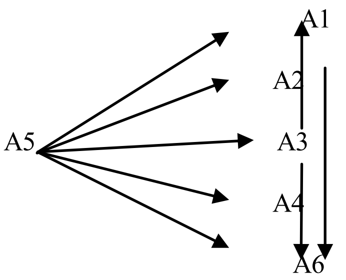
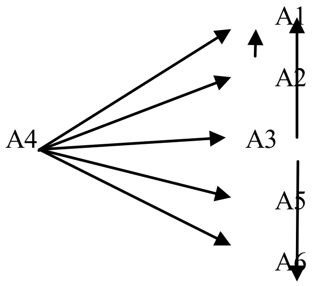
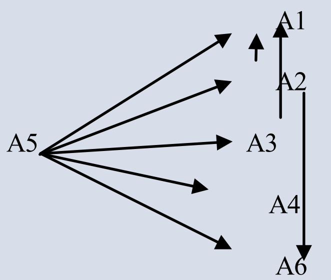
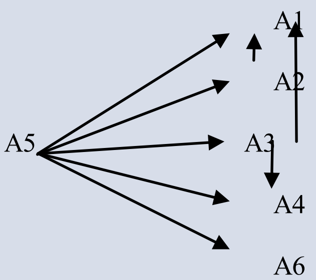
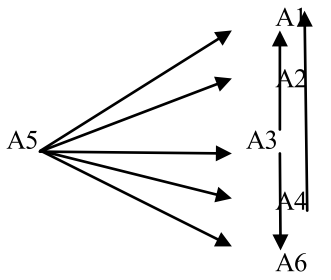
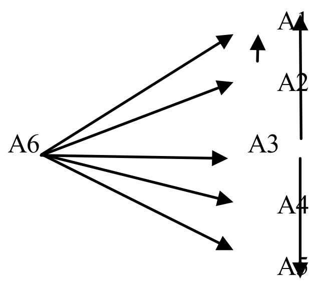
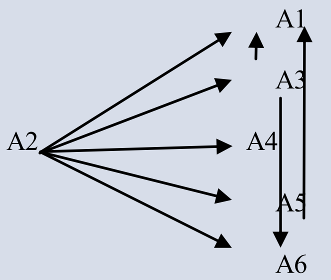
Format manipulated	*Priority*	*Format manipulated*	*Priority*
1,5		3,6	
1,6		4,5	
2,3		4,6	
2,4		5,6	
2,5		-	-

Table 12: AHP Model / Distribute Mode Results (change of alternatives format)

Format manipulated	*Priority*	*Format manipulated*	*Priority*
1,2	$A_5 > A_1 > A_3 > A_4 > A_6 > A_2$ .246 .215 .160 .157 .123 .099	2,6	$A_5 > A_6 > A_3 > A_4 > A_1 > A_2$ .246 .212 .160 .157 .130 .095
1,3	$A_5 > A_2 > A_1 > A_4 > A_6 > A_3$ .246 .184 .168 .157 .123 .122	3,4	$A_5 > A_4 > A_2 > A_1 > A_3 > A_6$ .246 .192 .184 .130 .125 .123
1,4	$A_5 > A_2 > A_3 > A_4 > A_1 > A_6$ .246 .184 .160 .153 .134 .123	3,5	$A_5 > A_2 > A_3 > A_4 > A_1 > A_6$ .224 .184 .182 .157 .130 .123
1,5	$A_1 > A_5 > A_2 > A_3 > A_4 > A_6$ .191 .186 .184 .160 .157 .123	3,6	$A_5 > A_2 > A_6 > A_4 > A_1 > A_3$ .246 .184 .166 .157 .130 .117
1,6	$A_5 > A_2 > A_3 > A_4 > A_6 > A_1$ .246 .184 .160 .157 .128 .126	4,5	$A_4 > A_5 > A_2 > A_3 > A_1 > A_6$ .214 .190 .184 .160 .130 .123
2,3	$A_5 > A_3 > A_4 > A_2 > A_1 > A_6$ .246 .206 .157 .137 .130 .123	4,6	$A_5 > A_2 > A_3 > A_4 > A_6 > A_1$ .246 .184 .160 .149 .131 .130
2,4	$A_5 > A_4 > A_3 > A_1 > A_6 > A_2$ .246 .238 .160 .130 .123 .103	5,6	$A_6 > A_2 > A_5 > A_3 > A_4 > A_1$ .188 .184 .182 .160 .157 .130
2,5	$A_5 > A_2 \approx A_3 > A_4 > A_1 > A_6$ .270 .160 .160 .157 .130 .123	-	-

Table 13: AHP Model / Ideal Mode Results (change of alternatives format)

Format manipulated	*Priority*	*Format manipulated*	*Priority*
1,2	$A_5 > A_1 > A_3 > A_4 > A_6 > A_2$ .878 .782 .587 .575 .488 .370	2,6	$A_5 > A_6 > A_3 > A_4 > A_1 > A_2$ .878 .780 .587 .575 .507 .354
1,3	$A_5 > A_2 > A_1 > A_4 > A_6 > A_3$ .878 .645 .632 .575 .488 .462	3,4	$A_5 > A_4 > A_2 > A_1 > A_6 > A_3$ .878 .693 .645 .507 .488 .469
1,4	$A_5 > A_2 > A_3 > A_4 > A_1 > A_6$ .878 .645 .587 .568 .514 .488	3,5	$A_5 > A_3 > A_2 > A_4 > A_1 > A_6$ .813 .652 .645 .575 .507 .488
1,5	$A_1 > A_5 > A_2 > A_3 > A_4 > A_6$ .698 .688 .645 .587 .475 .488	3,6	$A_5 > A_2 > A_6 > A_4 > A_1 > A_3$ .878 .645 .630 .575 .507 .445
1,6	$A_5 > A_2 > A_3 > A_4 > A_6 > A_1$ .878 .645 .587 .575 .505 .491	4,5	$A_4 > A_5 > A_2 > A_3 > A_1 > A_6$ .758 .695 .645 .587 .507 .488
2,3	$A_5 > A_3 > A_4 > A_1 > A_2 > A_6$ .878 .737 .575 .507 .495 .488	4,6	$A_5 > A_2 > A_3 > A_4 > A_6 > A_1$ .878 .645 .587 .551 .512 .507
2,4	$A_5 > A_4 > A_3 > A_1 > A_6 > A_2$ .878 .843 .587 .507 .488 .377	5,6	$A_6 > A_5 > A_2 > A_3 > A_4 > A_1$ .695 .671 .645 .587 .575 .507
2,5	$A_5 > A_3 > A_4 > A_2 > A_1 > A_6$ .963 .587 .575 .561 .507 .488	-	-

Table 14: AHP Model /Multiplicative Mode Results (change of alternatives format)

Format manipulated	*Priority*	*Format manipulated*	*Priority*
1,2	A5 > A4 > A1> A3 > A2 > A6 .496 .314 .099 .048 .042 .002	2,6	A5 > A4 > A6> A3 > A1 > A2 .496 .314 .099 .048 .042 .002
1,3	A5 > A4 > A2> A1 > A3 > A6 .496 .314 .099 .048 .042 .002	3,4	A5 > A3 > A2> A4 > A1 > A6 .496 .314 .099 .048 .042 .002
1,4	A5 > A1 > A2> A3 > A4 > A6 .496 .314 .099 .048 .042 .002	3,5	A3 > A4 > A2> A5 > A1 > A6 .496 .314 .099 .048 .042 .002
1,5	A1 > A4 > A2> A3 > A5 > A6 .496 .314 .099 .048 .042 .002	3,6	A5 > A4 > A2> A6 > A1 > A3 .496 .314 .099 .048 .042 .002
1,6	A5 > A4 > A2> A3 > A6 > A1 .496 .314 .099 .048 .042 .002	4,5	A4 > A5 > A2> A3 > A1 > A6 .496 .314 .099 .048 .042 .002
2,3	A5 > A4 > A3> A2 > A1 > A6 .496 .314 .099 .048 .042 .002	4,6	A5 > A6 > A2> A3 > A1 > A4 .496 .314 .099 .048 .042 .002
2,4	A5 > A2 > A4> A3 > A1 > A6 .496 .314 .099 .048 .042 .002	5,6	A6 > A4 > A2> A3 > A1 > A5 .496 .314 .099 .048 .042 .002
2,5	A2 > A4 > A5> A3 > A1 > A6 .496 .314 .099 .048 .042 .002	-	-

*. For cost criteria: Xij^{-Wj}.

Findings

1. Comparative results (from Tables 9-14, to table 2) indicated the stability of the TOPSIS, PROMETHEE, ELECTRE, and multiplicative mode of AHP results. In other words, the change in order of alternatives presentation has no impact on the results (rank preservation).
2. It is remarkable to observe that in this illustrative example, the distributive and ideal mode of AHP has very bad performance, and is the case of rank reversal phenomenon (Table 12 and 13; 11 of 15 cases for distributive mode, 12 of 15 cases for ideal mode, respectively). In other words, the change in order of alternatives presentation has a great impact on the results (rank reversal phenomenon).

Concluding Remarks

This paper discusses what happens if we change the format of alternative or attribute presentation in decision making matrix and redo our assessment of alternatives, in some of MADM models (i.e., TOPSIS, PROMETHEE II, ELECTRE, and three versions of AHP (distributive, ideal, and multiplicative modes)). The results showed that the stability of the TOPSIS, PROMETHEE II, and ELECTRE model results. Meanwhile, all the AHP variants (i.e., distributive, ideal, and multiplicative modes) have bad and unreasonable performance (rank reversal phenomenon). Even so, DMs (Decision Makers) priorities are affected if alternatives or attributes are added to or deleted from the set. It is unreasonable when introduce no new alternative or attributes.

Generally, despite the rigorous mathematical foundation of AHP and its variants, this notable limit existed. In addition, we suggested that further research should be done to confirm these observations and why this happened.

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