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The Impact of the Scale Elements Alteration on Priorities in Analytic Hierarchy Process Technique

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Abstract

The present study, presents a comparative analysis of different measurement scales adopted in Analytic Hierarchy Process (AHP), by testing them versus a problem with a known composite answers. Then experimentally, the impact of the different measurement scale elements alteration from three aspects: 1. The limited scale upper bound (up to 9), 2. Changing the scale parameters (a parameters), and 3. Changing the system numbers (from 1, 3…9; to 2, 4…10) on priorities are investigated. The results show that the linear measurement scale has the best performance in comparison to other scales.

Keyword: AHP, Measurement Scale, Scale Elements Alteration

Introduction

Analytic Hierarchy Process (AHP) has been a tool in the hands of decision makers and researchers since its invention; it is still the most widely used multi-criteria decision making method (Turskis et al., 2009). AHP is based on three basic principles: decomposition, comparative judgments, and hierarchic composition or synthesis of priorities. The decomposition principle is applied to structure a complex problem into a hierarchy of clusters, sub-clusters, sub-sub-clusters and so on. The principle of comparative judgments is applied to construct pair wise comparisons of all combinations of elements in a cluster with respect to the parent of the cluster. These pair wise comparisons are used to derive 'local' priorities of the elements in a cluster with respect to their parent. The principle of hierarchic composition or synthesis is applied to multiply the local priorities of elements in a cluster by the 'global' priority of the parent element, producing global priorities throughout the hierarchy and then adding the global priorities for the lowest level elements (the alternatives), (Forman and Selly, 2001). One of AHP's strengths is the possibility to evaluate quantitative as well as qualitative criteria and alternatives on the same preference scale of nine levels. These can be numerical, verbal, or graphical (Ishizaka and Labib, 2009). Theoretically there is no reason to get restricted only to these numbers. Therefore, other scales have been proposed (Ishizaka et al., 2011). Since, the main goal of the present research is to evaluate the different measurement scales and scale elements alteration on priorities in AHP techniques.

The paper is organized as follows. In section 2; the AHP, section 3; measurement scale and section 4; literature is reviewed. Numerical example is provided in section 5; the paper is concluded in section 6.

Analytic Hierarchy Process (AHP)

AHP is an intuitive method for formulating and analyzing decisions. AHP has been applied to numerous practical problems in the last few decades. Because of its intuitive appeal and flexibility, many corporations and governments routinely use AHP for making major policy decisions (Ramanathan, 2001). It is not the purpose of this paper to explain in detail the AHP methodology. See for instance Saaty (2000). A brief discussion of AHP is provided in this section.

The AHP method uses the pair wise comparisons and eigenvector methods to determine the a_{ij} values and also the criteria weights W_j. In this method; a_{ij} represents the relative value of alternative A_i when it is considered in terms

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of criterion C_j. In the original AHP method, the a_{ij} values of the decision matrix need to be normalized vertically. That is, the elements of each column in the decision matrix add up to 1. In this way, values with various units of measurement can be transformed into dimensionless ones. If all the criteria are benefit criteria (that is, the higher the score the better the performance is), then according to the original AHP method, the best alternative is the one that satisfies the following expression:

$$P^*_{AHP} = \underset{i}{\text{Max}}\ P_i = \underset{i}{\text{Max}} \sum_{j=1}^{n} a_{ij} W_j, \text{ for } i=1, 2, 3 \ldots m.$$

From the above formula, it can be seen that the original AHP method uses an additive expression to determine the final priorities of the alternatives in terms of all the criteria simultaneously (Wang, 2007). Generally, the purpose of the AHP is to assist people in organizing their thoughts and judgments to make more effective decisions (Saaty, 2000).

Measurement Scale

Almost all sciences use numbers. These numbers appear throughout all levels of the complex chain of mathematical, logical, and heuristic analyses that constitute scientific explanation and argumentation. Usually the first place they appear is in the quantification of empirical concepts. This step is usually called measurement (Narens, 1981). Measurement is any set of rules for assigning numbers that are attributed to objects (Saaty, 2004). The fact that numerals can be assigned under different rules leads to different kinds of scales and different kinds of measurements (Stevens, 1946). The only rule not allowed would be random assignment, for randomness in effect amounts to a non-rule (Luce, 1997). In his 1946 and 1951 publications Stevens singled out four groups of transformations on the real or positive real numbers as relevant to measurement: one-to-one, strictly monotonic increasing, affine, and similarity, and he introduced the corresponding terms of nominal, ordinal, interval, and ratio to refer to the families of homomorphism, or scales, related to these groups (Narens and Luce, 1986).

A commonly used measurement scales in the AHP is the ratio scale (Vachajitpan, 2004). Perhaps the most significant aspect of the AHP is in its use of ratio scales (Saaty, 2000). The measurement scale proposed by Saaty in the AHP is a 1 to 9 point scale. It is used to indicate the number of times one criterion is better than other criteria in the pair wise comparison. The reverse relationship is represented by an inverse of the assigned value. Thus, it is impossible to have a zero or a negative value in the AHP scale (Vachajitpan, 2004).

In a judgment matrix, instead of assigning two numbers W_i and W_j (numbers that generally we do not know), as one does with tangibles, and forming the ratio W_i / W_j we assign a single number drawn from the fundamental scale of absolute numbers shown in table 1, to represent the ratio $(W_i / W_j)/1$, (Saaty, 2005). Theoretically there is no reason to be restricting to these numbers. Therefore, other scales have been proposed (table 2), (Ishizaka et al., 2011).

Table 1: The fundamental scale of absolute numbers

Intensity of Importance	*Definition*	*Explanation*
1	Equal importance	Two activities contribute equally to the objective.
2 3	Weak Moderate importance	Experience and judgment Slightly favor one activity over another.
4 5	Moderate plus Strong importance	Experience and judgment Strongly favor one activity over another.
6 7	Strong plus Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice.
8 9	Very, very strong Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation.
Reciprocals of above	If activity I has one of the above non zero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i	A reasonable assumption.
Rationales	Ratio arising from the scale	If consistency were to be forced by obtaining n numerical values to span the matrix.

Ref. Saaty (2005), p. 356.

In general, evaluating of the impact of the different measurement scale elements alteration on priorities in AHP is the aim of this paper.

Table 2: Different Measurement Scales

scale	definition	parameters
linear	C=a.x	a>0;x=1,2,…,9
power	$C=x^a$	a>1;x=1,2,…,9
geometric	$C=a^{x-1}$	a>1;x=1,2,…,9
logarithmic	$C=\log_a^{(x+1)}$	a>1;x=1,2,…,9
Root square	$C=\sqrt[a]{x}$	a>1;x=1,2,…,9
Inverse linear	C=9/(10-x)	a>1;x=1,2,…,9
balanced	C=w/(1-w)	W=0.5,0.55,0.6,…,0.9

Ref. Ishizaka et al., (2011), p. 4.

Literature Review

In the current literature, there are several examples where different measurement scale is used and compared in the choice of the final solution. Here, we will mention some of them. Poyhonen et al., (1997) performed a comparative study in which subjects were requested to quantify verbal ratio statements by adjusting the heights of visually displayed bars. Salo and Hämäläinen (1997) applied multi attribute value theory as a framework for examining the use of pair wise comparisons in the AHP. Next, it is demonstrated that the AHP can be modified so as to produce results similar to those of multi attribute value measurement. Then, the new balanced scales to improve the sensitivity of the AHP ratio scales are proposed. Triantaphyllou et al., (1998) provided a comprehensive survey of some methods for eliciting data (measurement scale) for MCDM (Multiple Criteria Decision Making) problems and also for processing such data. Sato (2001) studied to find the scale (i.e. linear and power scale) of the AHP appropriate for representing decision maker's perception. Result indicated that the power scale is preferable to the linear scale as the judgment scale. In Shinohara et al., (2001) various methods, such as the eigenvector method, the geometric mean method, and the entropy method have been proposed to estimate a weight vector from pair wise comparison data. The results indicated that, when a decision maker decides each element of a pair wise comparison matrix on the basis of linear scale, the entropy method is expected to produce a weight vector that is closest to the true weight vector. On the contrary when a decision maker decision is on the basis of exponential scale, the eigenvector method and the geometric mean method are expected to produce weight vectors closer to the true weight vector. Vachajitpan (2004) developed a different model based on the least square principle to apply to situations where either the ratio or the intervals scaling method are used. Wedley (2007) studied the role of natural zero in scale for generating priorities. Monat (2009) proposed the use of global scales instead of local scales. (Because, using the local scales tends to overemphasize the small differences in attribute measures). Cox (2009) used a graph for interpreting of multidimensional data. So, at first the much dissimilarity generated in the ANP (Analytic Network Process) is analyzed using individual differences scaling. Secondly the single sets of dissimilarities that arise from the AHP are analyzed using multidimensional scaling. Ishizaka et al., (2011) demonstrated that the aggregation method of local priorities and the measurement scale in AHP has a strong influence on the selection of the compromise and therefore on the degree of concordance with the utility theory. Munshi (2014) proposed a method by which likert scales may be tailored for any given instrument and semantics. The method consists of performing a pretest using unmarked lines as scales, measuring the distances marked by the respondents, and using cluster analysis to determine the best placement of scale points for the actual study. However, as far as we know, a few experimental studies have addressed a fundamental problem discussed in this paper (altering the measurement scale elements). I.e. Triantaphyllou et al., (1994) used two evaluative criteria: 1. the ranking yielded when the CDP (the Closest Discrete Pair wise) matrix is used should not demonstrate any ranking inversions when the CDP ranking is compared with the ranking derived from the RCP (the Real Continuous Pair wise) matrix. 2. The ranking yielded when the CDP (the Closest Discrete Pair wise) matrix is used should not demonstrate any ranking indiscrimination when the CDP ranking is compared with the ranking derived from the RCP (the Real Continuous Pair wise) matrix, to examine a total of 78 scales which can be derived from two widely used scales (altering the two measurement scale parameters): 1. original scale (linear scale proposed by Saaty) and 2. Exponential scale. Results demonstrated that there is no single scale that can always be classified as the best or the worst scale for all cases. Ji and Jiang (2003) first reviewed and compared different scales from different aspects. Then discussed the transitivity of AHP scales and derived a scale based on the transitivity. Next, proposed two approaches for determining the parameter of the derived transitive scale. The result indicated that, proposed scale is quite simple and practicable. This paper proposes a new approach as discussed below.

Numerical Example

To derive priorities, the verbal comparisons must be converted into numerical ones (Ishizaka and Labib, 2009).

A comparison of the different measurement scales (based on the formula in table 2) is given in table 3.

Table 3: Different Scales for Comparing Two Alternatives

Scale type	Values								
linear	1	2	3	4	5	6	7	8	9
power	1	4	9	16	25	36	49	64	81
geometric	1	2	4	8	16	32	64	128	256
Logarithmic	1	1.58	2	2.32	2.58	2.81	3	3.17	3.32
Root square	1	1.41	1.73	2	2.23	2.45	2.65	2.83	3
Asymptotical*	1	0.12	0.24	0.36	0.46	0.55	0.63	0.70	0.76
Inverse linear	1	1.13	1.29	1.5	1.8	2.25	3	4.5	9
Balanced	1	1.22	1.5	1.86	2.33	3	4	5.67	9

Ref. Ishizaka and Labib (2009), p. 209.

*. Accordance Vachajitpan (2004), "it is impossible to have a zero or negative value in the AHP scale". Therefore, the asymptotical scale will not be discussed here.

In this section, we study how by altering the different measurement scale elements from the different aspects we can analyze the existing measurement scales. Experimentally, three tests include: 1. the limited scale upper bound (up to 9), 2. Changing the scale parameters (a parameters), and 3. Changing the system numbers (from 1, 3…9; to 2, 4…10) provided. Next, via numerical example the impact of different measurement scale on priorities and their performance in terms of each one of the test criterion are investigated. Here, two points are noteworthy. First, in accordance to Wedley (2001, p. 551):

> "At the 5th international symposium of the Analytic Hierarchy Process in Kobe Japan, Thomas Saaty suggested to the author that the controversy regarding correct synthesis modes for the AHP should be tested with problems with known true values …".

Second, in accordance to Saaty (2000, p. 455) have:

> "In the AHP one needs to be careful with criteria measured on the Same absolute scale. Criteria measured in dollars are a common Example of this. The priority of each criterion must be equal to The sum of the measurements of its alternatives divided by the Sum of the measurements of the alternatives with respect to all These criteria. Only then can one normalize the measurements Of the alternatives, weight them by these priorities and add to Obtain the relative weights of the alternatives with respect to All these criteria".

Since, the use of problems with known answers is the aim of this paper. To illustrate these basic ideas, and assuming that all of the criteria are expressed in the same unit, a simple 4.4 decision matrix is presented (table 4).

Table 4: Problem with Known Weights

Cri. / Alt.	C1	C2	C3	C4	total	True weights
A1	1	5	9	3	18	0.225
A2	7	3	1	9	20	0.250
A3	3	3	9	7	22	0.275
A4	5	7	3	5	20	0.250
total	16	18	22	24	80	-

In the absence of any other standards, the solution provided by this approach (A3, .275 > A2, .250 = A4, .250 > A1, .225), was used as the standard. Since, this could cause some bias in the final result.

Table 5: AHP Results for Linear Scale

Cri. / Alt.	C1 16/80 =.200	C2 18/80 =.225	C3 22/80 =.275	C4 24/80 =.300	Composite priorities
A1	0.063	0.278	0.409	0.125	0.225
A2	0.438	0.167	0.045	0.375	0.250
A3	0.188	0.167	0.409	0.292	0.275
A4	0.313	0.389	0.136	0.208	0.250

A comparison of the test results is given in table 6. i.e. for linear measurement scale (based on table 4):

Notes: According to axiom 3 of AHP, the criteria are assumed to be independent of the alternatives (Wedley, 2001). Here, we violated this property.

Table 6: AHP Results for Different Measurement Scales

Scale type	*Priorities (rank and intensity)*
linear	**A3 > A2 = A4 > A1** .275 .250 .250 .225
power	A3 > A2 > A1 > A4 .289 .273 .227 .211
geometric	A3 > A2 > A1 > A4 .318 .316 .269 .097
logarithmic	A3 > A4 > A2 > A1 .267 .263 .241 .230
Root square	A4 > A3 > A2 > A1 .263 .262 .241 .233
Inverse linear	A3 > A2 > A1 > A4 .292 .287 .263 .158
balanced	A3 > A2 > A1 > A4 .288 .279 .249 .183

Findings

1. Different measurement scales for deriving priorities, can lead to different results.
2. As seen from the table, the linear measurement scale results are the same (rank and intensity) as displayed in the last column of table 4 (standard).
3. It is remarkable to observe that in this illustrative example using the all of the scales, except Root square, the A3 ranking is best.

Test Criterion

The Limited Scale Upper Bound (up to 9)

In this section, the all of the measurement scales are upper bounds, by altering scale parameters restricted to the 9 (table 7). Because, the base (upper bound) for original AHP scales is nine. Then, the behaviors of these methods (scales) by their impacts on priorities are investigated.

Table 7: Modified Measurement Scales

Scale type	*Scale parameter*	*values*								
Linear*	-	1	2	3	4	5	6	7	8	**9**
power	a=1.0001	1	2	3	4	5	6	7	8	**9**
geometric	a=1.3161	1	1.32	1.73	2.28	3	3.95	5.20	6.84	9
Logarithmic	a=1.2910	2.71	4.3	5.43	6.3	7.02	7.62	8.4	8.6	**9**
Root square	a=1.0001	1	2	3	4	5	6	7	8	**9**
Inverse linear*	-	1	1.13	1.29	1.5	1.8	2.25	3	4.5	**9**
Balanced*	-	1	1.22	1.5	1.86	2.33	3	4	5.67	**9**

*. Does not require this modification.

i.e. for geometric scales:

Table 8: Initial Information by Geometric Measurement Scale (Based on table 3)

Cri. / Alt.	*C1*	*C2*	*C3*	*C4*
A1	1	16	256	4
A2	64	4	1	256
A3	4	4	256	64
A4	16	64	4	16

Table 9: Modified Information (Based on table 7, for geometric measurement scale)

Cri. / Alt.	*C1*	*C2*	*C3*	*C4*
A1	1	16	256	4
A2	64	4	1	256
A3	4	4	256	64
A4	16	64	4	16

Table 10: AHP Result (For Information with Modified Geometric Measurement Scale)

Cri. / Alt.	C1 .176	C2 .187	C3 .333	C4 .304	Composite priorities
A1	.091	.257	.434	.091	.237
A2	.476	.148	.048	.475	.272
A3	.158	.148	.434	.275	.284
A4	.274	.446	.083	.158	.208

A comparison of the test results is given in table 11.

Table 11: AHP Results for Different Modified Measurement Scales (table 7)

Scale type	Priorities (rank and intensity)
linear	**A3 > A2 = A4 > A1** .275 .250 .250 .225
power	**A3 > A2 = A4 > A1** .275 .250 .250 .225
geometric	A3 > A2 > A1 > A4 .284 .272 .237 .208
logarithmic	A3 > A4 > A2 > A1 .267 .263 .241 .228
Root square	**A3 > A2 = A4 > A1** .275 .250 .250 .225
Inverse linear	A3 > A2 > A1 > A4 .292 .287 .263 .158
balanced	A3 > A2 > A1 > A4 .288 .279 .249 .183

Findings

1. As seen from the table, linear, power, and root square scale results (after modification) are the same, as compared with standard.
2. Another importance point to observe is that, in the power and root square modified scales, priorities (rank and intensity) are changed (from: A3, .289 > A2, .273 > A1, .227 > A4, .211; for power scale, and A4, .263 > A3, .262 > A2, .241 > A1, .233; for root square scale, to: A3, .275 > A2, .250 = A4, .250 > A1, .225), and similar to standards. Therefore, the power and root square scales to scale parameters (**a**) are highly sensitive and differ from other scales.
3. It is remarkable to observe that in this illustrative example, from all of the modified measurement scales, the A3 ranking is best.

Changing the Scale Parameters (a parameters)

In this section, the change of the scale parameters from a=2 (except for linear scale; from a=1), to a=3 and a=5 for all scales, is considered (table 12).

Table 12: The Change of Scale Parameter Results

Scale type	Scale parameters	values								
Linear	a=3	3	6	9	12	15	18	21	24	27
	a=5	5	10	15	20	25	30	35	40	45
Power	a=3	1	8	27	64	125	216	343	512	729
	a=5	1	32	243	1024	3125	7776	16808	32768	59049
Geometric	a=3	1	3	9	27	81	243	729	2187	6561
	a=5	1	5	25	125	625	3125	15625	78125	390625
Logarithmic	a=3	.631	1	1.262	1.465	1.631	1.771	1.893	2	2.096
	a=5	.431	.683	.861	1	1.113	1.209	1.292	1.365	1.431
Root square	a=3	1	1.260	1.442	1.587	1.710	1.817	1.913	2	2.080
	a=5	1	1.149	1.246	1.320	1.380	1.431	1.476	1.516	1.552
Inverse linear*	a=3	-	-	-	-	-	-	-	-	-
	a=5	-	-	-	-	-	-	-	-	-
Balanced*	a=3	-	-	-	-	-	-	-	-	-
	a=5	-	-	-	-	-	-	-	-	-

*. Will not be examined. Because, different parameters (from **a** parameters) were used.

A comparison of the test results is given in table 13.

Table 13: Changing the Scale Parameter Results

Scale type	*Scale parameters*	*Priorities (rank and intensity)*
Linear	a=3	**A3 > A2 = A4 > A1** **.275 .250 .250 .225**
	a=5	**A3 > A2 = A4 > A1** **.275 .250 .250 .225**
Power	a=3	A3 > A2 > A1 > A4 .302 .295 .237 .166
	a=5	A3 > A2 > A1 > A4 .321 .320 .262 .098
Geometric	a=3	A3 > A2 > A1 > A4 .330 .329 .300 .041
	a=5	A2 = A3 > A1 > A4 .333 .333 .321 .014
Logarithmic	a=3	A3 > A4 > A2 > A1 .267 .263 .241 .230
	a=5	A3 > A4 > A2 > A1 .267 .263 .241 .230
Root square	a=3	A3 > A4 > A2 > A1 .261 .257 .245 .237
	a=5	A3 > A4 > A2 > A1 .257 .256 .246 .241

Findings

1. The results indicate that, only the linear measurement scale priorities in both scale parameters (a=3, 5) are the same as compared with standard.
2. As seen from the table, the geometric measurement scale results are shown different priorities for a=3 and 5.
3. Another important point to be observed is that, the A3 is ranking best for all the scales.
4. As seen from the table, the logarithmic measurement scale's result are showing same priorities for a=3 and 5. However, do not exhibit same priorities with standard.

Changing the System Numbers

Here, the switch from: 1-3-5-7-9 to: 2-4-6-8-10 values, is considered. A comparison of the test result is given in table 14.

Table 14: Changing the System Numbers Results

Scale type	*Priorities (rank and intensity)*
linear	A3 > A2 = A4 > A1 .271 .250 .250 .229
power	A3 > A2 > A1 > A4 .285 .267 .227 .221
geometric	A3 > A2 > A1 > A4 .318 .316 .269 .097
logarithmic	A3 > A4 > A2 > A1 .262 .258 .244 .236
Root square	A3 > A4 > A2 > A1 .262 .255 .246 .236
Inverse linear*	-
Balanced*	-

*. Will not be examined. Because, different parameters (from **a** parameters) were used.

Notes: the standard priorities for new situation (system 2-4-6-8-10), calculated as: A3, .271 > A2, .250 = A4, .250 > A1, .229.

Finding

1. The standard and linear measurement scale priorities, exhibit the same ranking, with slightly different intensities from the previous system (1-3-5-7-9).
2. The geometric measurement scale; however, do not exhibit same priorities with standard. Nevertheless, are shown the same priorities as previous systems (table 6).

Conclusion

In this paper, we are focusing on the scale element alteration from the three aspects: 1. the limited scale upper bound (up to 9), 2. Changing the scale parameters (**a** parameters), and 3. Changing the system numbers (from 1, 3…9; to 2, 4…10) and their impacts on priorities in AHP. The major findings are as follow:

1. Test Criterion 1

Test criterion 1 is showing that, the linear, power, and root square modified scale results are the same, as compared with the standard. Whereas, beforehand, only the linear measurement scale gave the correct answers.

2. Test Criterion 2

Test criterions 2 are showing that, only the linear measurement scale in both scale parameters (a=3, 5), are having the same results, as compared with standard.

3. Test Criterion 3

Test criterions 3 are showing that, with changing the system numbers, simultaneously the standard and linear measurement scale priorities changed. So that, the same ranking with different intensities was obtained.

Generally, the results have shown that, the linear measurement scales have the best performance (or stability) in compare to another scale.

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