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EDDY CURRENT LOSS IN FERROMAGNETIC PLATES OF FINITE THICKNESS, SUBJECTED TO TWO BOUNDARY CONDITIONS OF MAGNETIC FIELD AT THE SURFACE CREATED BY ALTERNATING MAGNETIZING FIELD/FLUX

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ABSTRACT

The paper presents normalized eddy current loss curve for ferromagnetic plates of finite thickness, giving variation of normalized loss, denoted as (loss factor)$_L$ with plate relative thickness, d/δ_L where d represents plate half thickness and δ_L signifies depth of penetration used under limiting nonlinear theory.

Graphical solution approach, ignoring the effect of harmonics of field quantities has been used to solve Maxwell's field equation for thin iron plates subjected to sinusoidal magnetizing force at the surface, commonly denoted as SHS condition. Step function B_1-H approximate has been used for the magnetic characteristic of the plate material; and the result of graphical solution represented in the form of normalized eddy current loss and power factor curves. The appended eddy current loss curve has been shown useful for predicting eddy current loss in thin iron plates of varied electrical and magnetic properties, subjected to SHS condition. The result of the predicted values of eddy current loss have been compared with the corresponding test results for core losses and the two results are shown to be in close agreement within engineering accuracy for plates with relative plate thickness $\geq$ 0.40.

The application of the normalized eddy current loss has been attempted for predicting eddy current loss in thin iron plates subjected to the other boundary condition of the magnetic field at the surface, that is, sinusoidal current density at the surface, denoted as SJS condition and the results obtained have been compared with the corresponding test results for core losses. Surprisingly it has been found that the two results agree with each other within ± 14 percent for much thinner plates with relative plate thickness $\geq$ 0.28.

As such, the core losses in thin as well as thick iron plates subjected to two boundary conditions of magnetic field at the surface, namely, SHS and SJS can be assessed in the design office from the normalized eddy current loss which embodies frequency, resistivity, surface magnetizing force, H_s, saturation flux-density, B_s and plate half thickness. The power factor curve, appended can be utilized for calculating the power factor of the exciting winding circuit reflected by the magnetic circuit.

Index Terms: *Normalized eddy current, Loss curve for ferromagnetic plates, Harmonics, SHS conditions.*

I. INTRODUCTION

During the last six decades, predetermination of eddy current loss in ferromagnetic plates of varied thickness, subjected to alternating magnetic field/flux has been one of the burning problems for researching physicists and electrical engineers [1 to 8 and several others]. The predetermined values of eddy current loss have been found within ± 15 percent of the corresponding core losses as per work of various authors [4,5,7]. As such, the core losses can be easily assessed in the design office for iron plates of different composition from the results of their predicted values of eddy current loss and design engineers and manufacturers can take the advantage of these assessments.

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II. POPULAR METHODS OF SOLVING FIELD EQUATION FOR CALCULATION OF EDDY CURRENT LOSS IN IRON PLATES

Two alternative methods of solving Maxwell's field equation which governs the field quantities and eddy current loss in iron plates, subjected to alternating magnetic field/flux are available in the literature. These methods take care of the non-linearity of B-H/B_1-H characteristic of the core material. These are as under.

A. Graphical Solution Approach of Solving Field Equation

Graphical solution approach of solving field equation, pioneered by Pohl[1]. In this solution method, the harmonics of the field quantities are ignored and it is presumed that the application of sinusoidal magnetizing force at the surface, creates sinusoidal magnetizing force of the same frequency at the inner layers of the plate and the flux density at any layer is obtainable from B_1-H characteristic of the core material where

Table 1 : List of symbols used with their significance

H,H'	Amplitude magnetizing force, A m and normalized value of H respectively.
H_s, H_c, H_n, H_{n+1}	Amplitude of H at the surface, H at the central plane, H at the n^{th} layer and the $n+1^{th}$ layer respectively.
B, B_1, B_s	Flux density, fundamental component of flux-density and saturation flux density obtained form B_1-H curve at 0.8 H_s.
Φ, Φ'	Flux per unit length, Wb/m and normalized value of Φ respectively.
φ, φ'	Phase angle between phasors H and Φ as well as H' and Φ' respectively.
d	Plate half thickness, m
δ_L	Depth of penetration used under limiting nonlinear theory.
d/δ_L	Relative plate thickness used under limiting nonlinear theory.
ρ	Resistivity, ohm-m
ω	Angular frequency of the forcing function.
B_{s1}	Saturation flux density, B_s used under limiting nonlinear theory.
LNL	Limiting nonlinear
P_{L1}	Eddy current loss per unit surface area, W/m^2 obtained by using graphical approach neglecting harmonics = {loss in thick plate x (loss factor)$_L$}
P_{L1}' or $(L.F.)_L$	(Loss factor)$_L$ used under LNL theory.
$(PF)_L$	Power factor of exciting winding under LNL theory.
$(L.F.)_A$	Loss factor used by Agarwal. [5].
P_A	Eddy current loss, W/m^2 used by Agarwal. [5]
P_A	Loss in thick plate of iron x $(LF)_A$, W/m^2
δ_A	Depth of penetration used by Agarwal. [5]
$(PF)_A$	Power factor used by Agarwal. [5]
d/δ_A	Relative plate thickness used by Agarwal. [5]
B_A	Flux density used by Agarwal = ¾ flux density obtained from B-H curve at H_s.

B_1 stands for the fundamental component of the flux-density. The graphical solution method is quite simple and fast as it solves a field problem, which involves the boundary conditions at the central plane and surface of a plate, as initial value problem. In this solution approach, if the values of magnetizing force, H, flux Φ and phase angle between phasors $\dot{H}$ and $\dot{\Phi}$, φ are known at any layer of the plate, these can be obtained layer by layer for the subsequent layers which are towards the surface by using simple difference equations, the latter being the split form of Maxwell's field equation. And the eddy current loss can be easily obtained from the values of phase-quantities either graphically or by solving graphical solution equations numerically. problem both for generators and demands.

B. Relaxation Technique / Finite Difference Schemes of Solving Field Equation

These techniques of solving field equation and obtaining eddy current loss in iron plates under the influence of alternating magnetic field are superior to graphical solution approach as these techniques take into account the effects of harmonics of the field quantities.

However, in large number of cases, the graphical solution approach ignoring the effects of harmonics have been found by authors [4,5,6] capable to give results for eddy current loss matching with the results of eddy current loss obtained by method considering the effects of harmonics. As such, the net effect of harmonics is found to be self compensating.

Subba Rao et. al [6] have done lot of work on iron plates of finite thickness and have found from the test results that for the same root mean square value of surface magnetizing force, H; the core losses in thin iron plate under SHS condition of magnetizing field may be about 60 percent more than these under SJS condition of the field where SHS stands for sinusoidal magnetizing force at the surface and SJS denotes sinusoidal current density at the surface; the latter being condition of sinusoidal flux. In order to verify the influence of these two surface boundary conditions of the fields on eddy current loss, authors evaluated eddy current loss in two thin iron plates of specific thickness for different values of surface H for SHS and SJS conditions by using relaxation technique and have compared these with the corresponding test results for core losses. The close agreement between the evaluated eddy current loss with the corresponding test results have been found by authors [6]. This gives a solid clue that the core losses in thin plates can be assessed from the results of eddy current loss. However, no further work has been done to develop normalized eddy current in thin iron plates subjected to both the surface boundary conditions of the magnetic field. This may be due to enormous amounts of computational effort needed for developing normalized loss solution. However, Lim and Hammond [7] have developed normalized eddy current loss curves for fairly thick iron plates subjected to SHS condition of magnetic field by using finite difference scheme. The developed normalized eddy current loss curves have been designated by authors as "universal loss chart". It can be used to predetermine eddy current loss in fairly thick plates. Authors have used Frohlich B-H approximate of the form:

$$B = \frac{H_s}{a_o + b_o H_s}$$

where a_o & b_o are Frohlich constants. Frohlich

B-H curve is more realistic as it can follow any B-H curve faithfully with proper choice of its constants. The computational efforts needed for finite difference scheme as well as for relaxation technique are quite heavy whereas graphical solution approach needs less computational effort. As such, it is considered as useful tool for developing the present paper.

III. GRAPHICAL SOLUTION APPROACH USED UNDER LIMITING NONLINEAR THEORY

Graphical solution approach is applicable for any configuration of B_1-H curve. However, under the working condition of magnetic field, plates of iron are subjected to under high degree of saturation hence some authors [4,6] have used limiting nonlinear B_1-H characteristic for obtaining solution of the field equation under limiting nonlinear theory. The graphical solution approach yields results for field quantities and eddy current loss at faster rate in comparison to the sophisticated solution approach such as finite difference scheme and relaxation technique as in the former method, one simple graphical solution yields results for various thickness of iron plates. Using graphical solution approach, ignoring the effects of harmonics of the field quantities, Rajgopalan et al [4] have developed normalized eddy current loss curve for thick iron plates showing variation of normalized eddy current loss with normalized value of surface H, for step function B_1-H curve of magnitude B_s where B_s is the saturation flux density. Authors have also developed expressions for depth of penetration, power factor and eddy current loss per unit surface area under limiting nonlinear theory as under:

Depth of penetration, $\delta_L = 2.06\left[\dfrac{H_S\rho}{\omega B_S}\right]^{1/2}$ .... (1.1)

Power factor for thick plate, $(PF)_L$

thick = 0.8165 .... (1.2)

Eddy current loss per unit surface area,

P_{L1}thick = $H^2_S\rho/\delta_L$ W/m^2......... (1.3)

Where H_S = maximum value of surface force, A/m

ρ = resistivity of core material, ohm-m

ω = angular frequency of the forcing function

$B_S = B_L$ = saturation flux density obtained from B_1-H curve at 0.8 Hs. Here the factor 0.8 takes into account the effect of attenuation of flux density wave.

Authors [4] have found that the core losses under the two surface boundary conditions of the filed remain same for the same r.m.s. value of the surface H. Further the results of empirical formula for eddy current loss developed by authors have been found in close agreement with the corresponding core losses. Subba Rao [6] have pursued the graphical solution approach for solving the field equation for different values of H at the central plane, Hc and have presented their solutions for the field quantities in the form of three sets of curves showing the variation of normalized values of H at the surface, H'_S, flux Φ'_S and phase angle between phasors, $\dot{H}'_S$ and $\dot{\Phi}'_S$ denoted as φ'_S against plate half thickness, d with H'_c as parameter where H'_c represents normalized value of H at the central plane of the plate.

These three sets of curves are shown useful to predetermine the values of plate eddy current loss and power factor of the exciting winding.

Expression for eddy current loss :

$P_{L1} = 0.5\omega H_S\Phi_S \sin\varphi_S$ W/m^2

And power factor = Sin φ_S

It is worth mentioning that Agarwal [5] has developed analytical relationships for depth of penetration, δ_A, power factor, $(PF)_A$ and eddy current loss, P_A for thick as well as thin iron plates subjected to SHS condition of the field. The work of Agarwal [5] has been based on certain concept of mechanism of magnetizing field at the surface. This mechanism has been developed by Mc Clean [2] and Mc Connel [3]. And it has been found that the empirical relationships developed for eddy current loss and power factor give very satisfactory results for iron plats subjected to SHS condition. However, the concept cannot be extended for plates for finite thickness subjected to SJS condition as it leads to erroneous results for power factor and losses. However the valuable test results of Agarwal [5] for iron plates of varied thickness subjected to SHS condition has been liberally used in the present paper for comparison.

IV. A FEW WORDS ON THE PRESENT WORK

The present work on thin iron plates subjected to sinusoidal magnetizing force, at the surface, is an extension of the graphical solution approach followed by Subba Rao et al [6]. The pattern of normalization for H, Φ and φ used in the present work is same as that used Subba Rao [6].

It is shown that if one graphical solution is obtained for one value of H_C it is capable to give results for normalized loss. This has been shown possible by using d/δ_L as parameter in place of d. The normalized loss curve developed in this paper presents variation of normalized loss denoted as (loss factor)$_L$ with the relative plate thickness, d/δ_L. The loss curve is useful to predetermine eddy current loss in thick as well as thin plates subjected to SHS condition for relative plates thickness ≥ 0.40. In the paper it is shown that the normalized loss curve has better utility for plates subjected to SJS condition of magnetic field.

It is shown even for thinner plates with relative thickness much less than 0.40 the eddy current loss can be predetermined. Thus the normalized curve presented in the paper is shown useful for giving results of eddy current loss for both the conditions of surface magnetizing field. Of course, there is some restriction over the range of relative thickness of plates for its utility.

Further it is shown that the predetermined values of eddy current loss are within 15 percent of the core losses. As such, the core losses can be assessed from the result.

V. THEORY

Development of normalized eddy current loss curve for ferromagnetic plates of finite thickness subjected to sinusoidal magnetizing force at the surface. Before proceeding for developing graphical solution for plate of finite thickness let us distinguish between thick and thin plates (plate of finite thickness). Under limiting nonlinear theory a plats is denoted as thick plate, of its relative plates thickness, d/δ_L is more than or equal to unity where d denotes plate half thickness, δ_L represents depth of penetration, the letter has been defined by equation (1.1). A plate comes under the category of thin plate when its relative thickness is less than unity. When such thin plate is subjected to sinusoidal magnetizing force at the surface, H_C, the central value of magnetizing force has some finite value and flux, Φ_C and phase angle between $\dot{H}_C$ and $\dot{\Phi}_C$ phasors, φ_C are both zero, whereas in the case of thick plate when the plate is subjected to SHS condition, both H_C, as well as Φ_C vanish and φ_C is also zero.

A. Graphical Solution for Field Equation, for Plate of Finite Thickness, ignoring the effect of harmonics

Graphical solution approach of solving Maxwell's field equation, which governs the field quantities and eddy current loss in an iron plate, has been pioneered by Pohl [1]. The method is based upon certain assumptions that sinusoidal magnetizing force at the surface creates sinusoidal H at the inner layers of the plat and the method ignores the effect of harmonics of the field quantities. As such, the flux density at any layer is obtainable from B_1-H relationship where B_1 stands for fundamental component of B. It is further assumed that the core material is of homogeneous structure and is of isotropic nature. Using these assumptions, if the value of H, flux Φ and phase angle between phasors $\dot{H}$ and $\dot{\Phi}$, φ are known these can be obtained graphically or by numerical solution at other layers which are away from the central plane by using the difference equations which represent the split from of Maxwell's equation. These difference equations are as under:

$$\dot{H}_{n+1} = \dot{H}_n + \Delta\dot{H}_n = \dot{H}_n + j\frac{\omega}{\rho}\Phi_n \Delta_n \quad \text{.........} (2.1)$$

$$\dot{H}_{n+1} = \dot{H}_n + \Delta\dot{H}_n = \dot{H}_n + j\frac{\omega}{\rho}\Phi_n \Delta_n \quad \text{.......} (2.2)$$

and

$\varphi_n = sin^{-1}$ *(Power facto)r* .... (2.3)

Where H_n, Φ_n and φ_n are the values of magnetizing field, flux and phase angle at the n^{th} layer respectively. The suffix (n+1) is used for the values at $n+1^{th}$ layer. B_n represents the value of flux density B_1 at n^{th} layer and $\dot{B}_n$ is in phase with $\dot{H}_n$. For the starting layer (Central layer) suffix n has the value zero. When a graphical construction of H and Φ is to be obtained H_C or H_O is given some finite value and Φ_0 and φ_0 are both zero. And △x, the layer thickness is given some suitable value. By giving different values to n, commencing from 1 to higher values serially the phase quantities H, Φ and φ are obtained for various plate thickness, say, for n=N, the values of H at the surface, $H_S = H_N$, flux, $\Phi_S = \Phi_N$ and phase angle, $\varphi = \varphi_N$ and the eddy current loss can be obtained by using the relationship:

Eddy current per unit surface area =

$P_{L1} = 1/2\omega H_S\Phi_S\sin\varphi_S$ W/m² $= 1/2\omega H_N\Phi_N\sin\varphi_N$ W/m² ... (2.4)

plate half thickness = d = $\sum_{0}^{N}\Delta x$

Corresponding relative plate thickness

$$\frac{d}{\delta_L} = \frac{\sum_{0}^{N}\Delta x}{2.06\left(\frac{H_S\rho}{\omega B_S}\right)^{1/2}} \quad \text{..............................} (2.5)$$

B. Improvement on graphical solution for thin iron plates

Subba Rao et al [6] have developed graphical solution for thin plates of iron using step function B_1-H approximate of magnitude, B_S, the saturation flux density. Authors have chosen the values of B_S, resistivity, ρ and angular frequency, ω each of unity. As such, the normalized value of H, denoted as $H' = \frac{H}{\frac{\omega}{\rho} B_S} = H$ and normalized value of flux, Φ, $\Phi' = \frac{\Phi}{B_S} = \Phi$ and normalized value of φ, $\varphi' = \frac{\varphi}{1} = \varphi$.

As such, the surface value $H'_S = H_S$ (=$H`_n$) and $\Phi'_S = \Phi_S$ and $\varphi'_S = \varphi_n$. Authors [6] solved the field equation for multiple values of H'_C and obtained values of H'_S, Φ'_S and φ'_S for different values of plate half thickness. The results of the multiple solutions have been presented as three sets of curves showing variations of normalized value of H_s flux, Φ_S and φ_s against plate half thickness d with H'_C as parameter. These sets of curves have been shown useful to predict eddy current loss in plates of finite thickness by using the equation (2.4).

In the present work, same pattern of normalization is used for phasors $\dot{H}$, $\dot{\Phi}$ and phase angle φ but it shown that if d/δ_L is used as parameter in place of d, a simple graphical solution used for one finite value of H'_C and continued for large number of layers to cover the complete useful range of plate relative thickness , the eddy current loss for plate of finite thickness can be obtained and multiple solutions as used by earlier authors [6] are not needed. In order to justify the above statement, the following procedure is adopted. Suppose one proceeds to develop two or more graphical constructions for different values of H'_C and obtain the values of H'_S, Φ'_S, φ'_S, d and $\frac{d}{\delta_L}\left(= \frac{d}{2.06\sqrt{H'_S}}\right)$ at different plate thickness d. And after obtaining these values a curve is drawn between H'_S versus d/δ_L with H'_C as parameter, it is found that all the curves coalesce. It indicates that H'_S is independent of the value of H'_C and is a unique function of d/δ_L. In a similar way, it can be shown that Φ'_S as well as φ'_S are unique functions of d/δ_L.

Now at this stage let us introduction an other parameter for eddy current loss which is denoted as (loss factor)$_L$ or P'_{L1} is defined as the ratio of eddy current loss in thin plate obtained graphically to eddy current loss in the thick plate of iron of same material for the same value of surface H.

Thus the normalized value of eddy current loss

$$(\text{Loss factor})_L = P'_{L1} = \frac{0.5\omega H'_S \Phi' \sin\varphi'_S}{H'^2_S \frac{\rho}{\delta_L}}$$

$$= \frac{0.5 H'_S \Phi'_S \sin\varphi'_S}{\left(H'^2_S\right) / 2.06\sqrt{H'_S}}$$

$$= 1.03 \frac{\Phi'_S}{\sqrt{H'_S}} \sin\varphi'_S \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.6)$$

As (Loss factor)$_L$ is a function of Φ'_S, H'_S and φ'_S, it is also unique function of d/δ_L.

C. Procedure for Obtaining Normalized Loss Curve for Plate of Finite Thickness.

Under limiting nonlinear theory, step function B_1-H curve is used of magnitude B_S. The value of B_S, ω, ρ are each chosen of unity value. Quite low values of Δx and H'_C are chosen as 0.25×10^{-5} and Δx = 0.000005 and $\Phi'_C = 0$, $\varphi_C = 0$, the graphical solution is obtained for large number of layers and at each layer the values of H'_S, Φ'_S, φ'_S, power factor, d/δ_L and (Loss factor)$_L$ are obtained. As the values of H' and Φ' build up the value of Δx can be increased suitably.

The results of this graphical solution are presented in Table (1) and (2). Table (3) gives the value of (Loss factor)$_L$ and (Power factor)$_L$ for different values of relative plate thickness, d/δ_L. The fig. (1) appended shows the variation of (Loss factor)$_L$ with

d/δ_L. The fig (1) also gives the variation of power factor of the exciting winding with d/δ_L.

D. Procedure for Predicting Eddy Current Loss

(a). First the value of B_S the saturation flux density is obtained from B_1 – H curve at 0.8 H_S. The factor 0.8 takes into account the effect of attenuation of the flux density wave. Alternately B_S can be obtained from the Frohlich relationship for B_1-H curve as under:

$$B_S = \frac{0.8H_S}{(a + b \times 0.8H_S)}$$ where a and b are Frohlich constants.

(b) From the value of B_S, d/δ_L is calculated where d = plate half thickness and δ_L is obtained from equation (1.1)

(c) The value of (Loss factor)$_L$ is obtained for the value of d/δ_L obtained. Hence the eddy current loss can be calculated by using equation:

Eddy current loss = (Loss factor)$_L$ x loss in thick plate.

$$= (\text{Loss factor})_L \times \frac{H^2{}_S \rho}{\delta_L} \ \text{W/m}^2$$

III. RESULTS AND DISCUSSION

The predetermined values of eddy current losses are compared with the test results of core losses in plates subjected to SHS condition of magnetic field of Ref [5] and the predicted values of eddy current loss in plates subjected to SJS condition of Ref. [6]. These are appended in various Tables. Tables (4), (5) and (6) of ref. [5] for SHS condition of magnetization and Table (7) and (8) of Ref. [6] are for plates subjected to SJS condition.

The predetermined values of eddy current loss in plates subjected SHS condition are appended in Tables (4), (5) and (6) comparison shows that the predicated values are within ± 15 percent for plate relative thickness ≥ 0.40 Tables (7) and (8) indicate that the results of eddy current loss obtained from the normalized loss curve for plates subjected to SJS condition are within 13 percent of the test results for thinner plates with d/d_L ³ 0.28. As such, the normalized loss curve has better utility under this condition of magnetic field.

IV. CONCLUSION

From the comparison of results for SHS condition it can be concluded that for $d/\delta_L \geq 0.4$ the effect of harmonics of the field quantities is ignorable. However for thinner plates one has to consider the effect of third and fifth harmonics to obtain correct values of eddy current loss. For SJS condition of magnetic field the effect of harmonics is of self-healing type for much thinner plates. From the results of eddy current loss the core losses can be easily assessed.

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