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OPTIMIZING ACF USING WIENER-KHINCHIN THEOREM

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ABSTRACT

Since the past few decades, several applications have been using auto correlation as their prime function for various objectives. Here the focus is to optimize the auto correlation by reducing the buffer size and computation complexities while maintaining the same nature of auto correlation function, so that the processing gets better and faster. A relatively less researched approach for computing auto correlation function by following Wiener-Khinchin Theorem has been explored. In order to prove the robustness of the method, statistical analysis based on mean square error between the traditional method and auto correlation function using Wiener-Khinchin theorem method have been discussed. Moreover, a novel formula to find out the frequency resolution for auto correlation function under certain conditions also has been keyed out.

Keywords: *Auto correlation function, Wiener Khinchin Theorem, frequency resolution, mean square error.*

1. INTRODUCTION

Auto Correlation Function (ACF) has various applications in the field of signal and image processing. Basically, an auto correlation is the computation of similarity between the signal with its delayed form.

The normalized ACF is an effective tool for determining relative muscle activation attacks for postural control along objective quantification of muscle co activation, and relating muscle activations with mechanical events [1]. A study of intestinal motility in rabbits, which is very complex, is performed by auto-correlation methods [2]. Moreover, ACF is a powerful tool in the applications of satellite imaging which helps to describe the local texture features [3]. The properties of Huffman sequence of normal and complementary levels, which improves the performance in radar applications, can be determined using ACF [4]. The statistical property of ACF of an impulse response is used for analyzing the behavior of a radio channel [5]. Auto correlation on ultrasonic Doppler signals is used to determine pregnancy in sheep [6]. There have been cases where the power spectrum and the fundamental frequency estimation, which is found using auto correlation together, is used for applications in robot auditory sensor, robust speech recognition, an equalizer system enabling bass and treble controls for separate source and indexing of music for music retrieval system [7]. The auto correlation transform is used to optimize and synthesize combinational logic for Binary Decision Diagrams and to compute the function's complexity even when the information is not given about the function's use [8]. Even for surface roughness access in micro-scale and nano-scale region, auto-correlation is an important task to determine surface irregularities with respect to their periodicity and randomness [9].

ACF can be used to determine fundamental period and pitch detection[10]. Fundamental period can also be extracted even by using the short-time average magnitude difference function (AMDF). But when compared to ACF, AMDF method is found to prostrate towards errors [11]. Overall, in time domain analysis, the ACF method can be considered as a better method. Here the focus is towards a relatively less researched and utilized approach for computing autocorrelation following Wiener-Khinchin (WK)

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theorem. The forthcoming sections describes about traditional ACF and the new method for computing ACF along with various cases of mean square error between the two methods to prove its credibility.

2. ACF

ACF is a mathematical tool for determining the fundamental period in a signal. For a continuous real sequence $f(\tau)$, autocorrelation is given as

$$\mathrm{r}\,(\mathrm{t}) = \frac{1}{2\mathrm{T}} \int_{-\infty}^{\infty} f(\tau) f(t+\tau) d\tau \tag{1}$$

For a discrete sequence r[k], the autocorrelation function is given as,

$$r[k] = \sum_{i=0}^{N-1} f[i] f[i+k] \tag{2}$$

Autocorrelation of a sequence is maximum at, t=0 in equation (1) and k=0 in equation (2). As the correlation is taken for the same section of the waveform, equation (1) is represented as,

$$\int_{0}^{\infty} f(\tau) f(\tau+t) d\tau \leq \int_{0}^{\infty} f^{2}(\tau) d\tau \tag{3}$$

and equation (2) is represented in discrete form as

$$\sum_{0}^{N-1} f(i) f(i+k) \leq \sum_{0}^{N-1} f[i]^{2} \tag{4}$$

The R.H.S of above equation is for ACF at initial conditions. The similarity between the waveform is reduced as the waveform is shifted over the sample space which results in decrease of amplitude at the auto correlation output. When the wave is completely out of phase, the output of the autocorrelation is minimum. When the wave is further delayed or shifted, it matches in phase and results in a value ideally equal to the peak at the origin. The period between two consecutive peaks of auto correlation output gives us the fundamental period of the input signal. For computation of ACF of width N, the buffer size requirement is 2N and for computation, it needs N^2 multiplications and N (N-1) additions. The autocorrelation discards phase information, returning only the power, and is therefore an irreversible operation. The disadvantage of ACF is that since it uses N buffer size, for determining fundamental period, input frequencies which are smaller than $\frac{F_s}{N-1}$ cannot be found.

3. ACF USING WK THEOREM

Autocorrelation is related with Fourier transform as per Wiener-Khinchin theorem [12]. It states the autocorrelation function is the Fourier transform of the power spectrum.

$$r_{xx}(l) \xleftrightarrow{F} S_{xx}(\omega) \tag{5}$$

where $r_{xx}(l)$ is the autocorrelation of signal x for a shift of l and S_{xx} is the energy density spectrum of the signal x. Equation (5) shows that energy spectral density is the Fourier transform of its auto correlation sequence. This means that autocorrelation sequence of a signal and its energy spectral density contains the same information about the signal.

Equation 5 can be split into two steps as

$$\left|X(\omega)\right| = \left|\sum_{n=0}^{\infty} x[n] e^{-j\omega n}\right| \tag{6}$$

$$\mathrm{r[n]} = \frac{1}{N} \sum_{n=0}^{\infty} \left|X(\omega)\right| e^{j\omega n} \tag{7}$$

Here for computing ACF, the WK theorem is implemented by taking the Fourier transform of the signal and the inverse Fourier transform of its magnitude. Equations (6) and (7) show WK theorem for one dimension functions and hence the Fourier Transform is taken, otherwise for multi-dimension functions Fourier Bessel transform can be considered [13]. Generalization of WK theorem can be done for both random and deterministic signals using an arbitrary basis set [14]. The radix-2 FFT algorithm requires $\frac{\mathrm{N}}{2}\log_2 \mathrm{N}$ multiplications and $N\log_2 N$ additions. Here, Fourier Transform of the signal and inverse Fourier transform of its magnitude is calculated. Hence, the computations would be $N\log_2 N$ multiplications and $2N\log_2 N$ additions when radix-2

Fast Fourier Transform is concerned. Other than that there would be additional 2N multiplications and N additions for calculation of magnitude for Fourier transform of the signal. Hence the total multiplications and additions are $2N+N\log_2 N$ and $2N\log_2 N+N$ respectively. The buffer size required is N samples.

4. FREQUENCY RESOLUTION

The frequency resolution of ACF is different from that of frequency spectrum of Discrete Time Fourier transform (DTFT). By definition, frequency resolution is the ration of sampling frequency by total number of samples. Hence, the frequency resolution for DTFT is given by Fs/N, where N is the total samples and its spectrum is spaced by Fs/N values. The frequency resolution for DTFT is linear in nature. But for ACF, this reasoning does not work out as its frequency resolution is non-linear in nature. In The resolution of frequency below its fundamental period is given by

$$Fr_{dec}=\left(\left(\frac{Fs}{fp}\right)-\left(\frac{Fs}{fp+1}\right)\right) \tag{8}$$

and frequency resolution for increasing the fundamental period is found to be

$$Fr_{inc}=\left(\left(\frac{Fs}{fp+1}\right)-\left(\frac{Fs}{fp}\right)\right) \tag{9}$$

Here Fs is the sampling frequency and fp is the fundamental period. When sampling frequency is constant and linear fundamental period is maintained, the values are found to be non linear in nature. The following table shows the frequency resolution for various fundamental periods. The sampling frequency is 8000Hz for all the input frequencies taken, i.e. 20Hz, 33 Hz and 37 Hz.

Table 1 : Frequency resolution for various input frequencies

Input frequency	Fr_{dec}	Fr_{inc}
20	0.050125	0.0498753
33	0.136688	0.1355657
37	0.171920	0.1703371

5. RESULTS

After various tests and statistical analysis, various results were obtained. The following section regarding computation complexity and mean square error will show the various tradeoff's for computing ACF by applying WK theorem.

5.1 Computation Complexity

Computations between the tradional ACF and ACF as per WK theorem were done for diverse buffer size in order to find the credibility of the method. Input signal of varying sine wave frequencies with noise were considered. It was found that the proposed method had better computation complexity than the traditional method which is shown in the table given below. It is seen that for the buffer size of 2000 samples, the computation is reduced around 150 times and when buffer size is 4000, the computation is reduced around 300 times.

Table 2 : Comparison of computation between traditional ACF and ACF using WK

Buffer Size	Traditional ACF	ACF using WK
2000	4,000,000	25932
4000	16,000,000	55864
8000	64,000,000	119726

5.2 Mean Square Error

The mean square error is taken between the ACF output signals and the ACF based on WK theorem. Classically, MSE assumptions go wrong for various applications [15]. Here no such problem exists as the input signal is not re-ordered under any case and a huge amount of precise data is used for calculation; hence there is no damage in the measurements. Four cases were considered for obtaining clear results.

Case 1: Constant buffer size of 4000 samples.

Here the input frequency is taken from 20Hz to 120Hz along x-axis and sampling frequency from 1000 to 8000Hz along y-axis. MSE has high spikes initially for less input frequency and sampling frequency and as both increases, the wave looks periodic in nature. This is the same case for having buffer size without noise. The three dimension graph is shown in figure a and b.

Case 2: Constant input frequency 20Hz.

The sampling frequency is varied from 1000Hz to 7000Hz along x-axis and buffer size from 400Hz to 16000 Hz along y-axis. For small sampling frequency and large buffer size, mean square error is very high (above 50%). For smaller buffer length, MSE has spike nature. Here overall, MSE looks periodic in nature. The graphs are shown in figure c and d.

Case 3: Constant input frequency of 37 Hz.

The conditions of sampling frequency and the buffer size are same for 20 Hz input frequency. There are lots of cluster around 2500Hz to 4000Hz and for buffer size of around 10000 to 12000 Hz. For small sampling frequency, there is around 30 to 40 percentage of mean square error. Mean square error is more predominant either for combination of high sampling frequency and low buffer size or for low sampling frequency and large buffer size. The mean square error is very small for large sampling frequency and large buffer size. The graphs are shown in figure e and f.

Case 4: Constant Sampling Frequency 4000Hz

Here the sampling frequency is maintained constant but the input frequency and the buffer size is varied from 20Hz to 120 Hz along the x-axis and 400 to 16000 samples along y-axis. Here the mean square error is periodic in nature under any case. The graph in the top view position is shown in figure g and h.

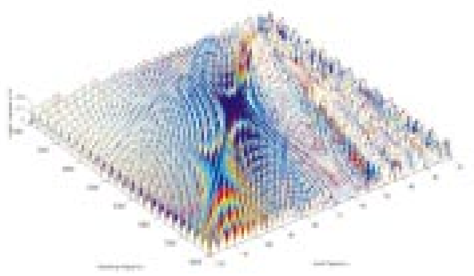

Figure a : Constant buffer length without noise

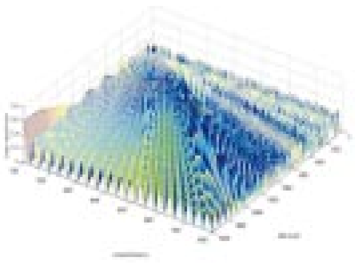

Figure b : Constant buffer length with noise

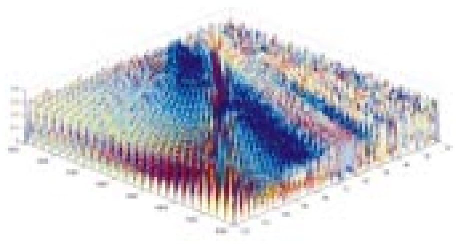

Figure c : Input frequency of 20Hz without noise

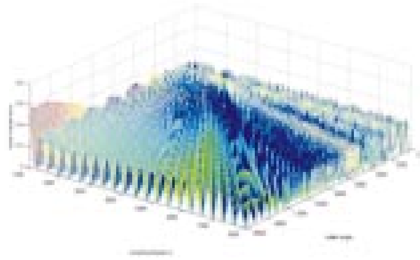

Figure d : Input frequency of 20Hz with noise

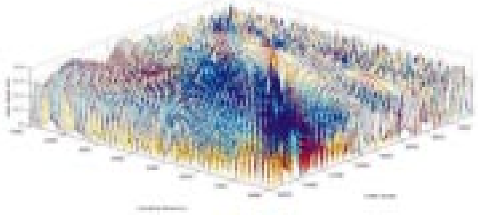

Figure e : Input frequency of 37Hz without noise

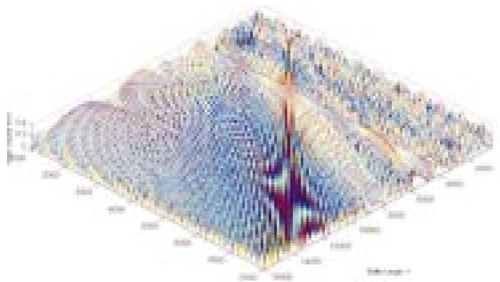

Fig f : Input frequency of 37with noise

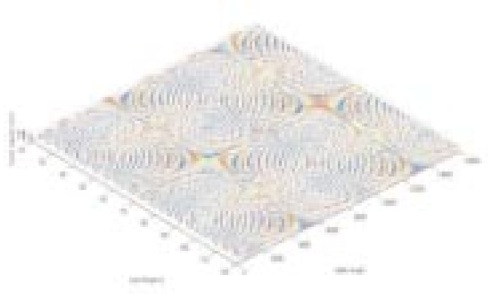

Figure g : **Sampling frequency of 4000Hz without noise**

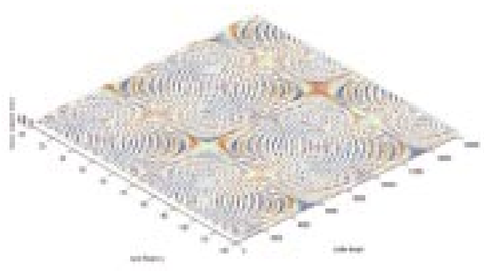

Figure h : **Sampling frequency of 4000Hz with noise**

6. RESULTS

It is noted that the buffer size requirement and the computation complexities for computing autocorrelation by WK theorem is very less compared to direct auto correlation method. When compared for the buffer size of 2000 samples, the computation is reduced around 150 times and when buffer size is 4000, the computation is reduced around 300 times. Here the radix-2 algorithm is applied for computation of Fourier transform, if higher radix algorithm is used, then the computations can be further reduced. Hence, by using ACF using WK, the computations are reduced drastically. ACF method is better for computing fundamental period as even small changes in peak difference gives different fundamental period because of non-linear nature of frequency resolution. The mean square error is high for small sampling frequency even when it exceeds nyquist crieterion and very low for large input and sampling frequency. Hence a good sampling frequency far better than nyquist criterion would be highly effective. Even for low buffer size the mean square error is huge around 40 to 50 percentage. In all, the mean square error values ranges from 0 to 10 percentage and under all the cases, mean square error looks periodic. Overall, in general applications this theorem works perfectly and only when special cases like low buffer size and sampling frequency, small changes from traditional method occurs.

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