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Retailer's Optimal Credit Period and Ordering Policy for Non-instantaneous Deteriorating Items with Trapezoidal Type Demand

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Abstract

It has been observed that items like trendy goods, cellular phones, electronic toys and others have trapezoidal type demand pattern and have a short life cycle due to competitive products. As a result, to boost the demand of an item, retailer offers a credit period to the customers. This might lead to default risks for the retailer. Moreover, for such items, non-instantaneous deterioration is observed. In this article, an optimal credit period and ordering policy are presented for items having trapezoidal type demand rate depending upon credit period offered to the customers; having non-instantaneous deterioration and no shortages. The objective is to maximize total profit function for an inventory system. Model is supported with numerical examples and sensitivity analysis is carried out.

Keywords: Inventory System, Non-Instantaneous Deterioration, Trapezoidal Demand, Credit Period Policy.

Classification: 90B05

Introduction

In the current worldwide and competitive market, retailer offers a delay period without any interest charges to attract more customers on the purchase made with support of some other financial companies. This results into increase in sale. On the other hand, this delay period offer leads to a default risk for the retailer when the customer declares that he is unable to pay back. It is clear that default risks are in direct proportion with the length of the delay period. Goyal (1985) gave an inventory model with permissible delay in payments. Shah et al. (2010) carried out review article on inventory systems with trade credits. Shah et al. (2014) discussed optimal ordering and purchase quantity model when demand depends upon credit period offered.

At present, the effect of deterioration on items cannot be ignored in an inventory system. Ghare and Schrader (1963) made the first attempt to integrate effect of deterioration in inventory system. Shah and Shah (2000), Goyal and Giri (2001), and Bakker et al. (2012) cite up-to-date review on deteriorating inventory system. However, most of researches assumed that the deterioration takes place as soon as the replenishment of an item in inventory system. In reality, most commodities maintain their quality or original conditions over a span, i.e., during this span, deterioration does not occur. Normally, it is observed that foodstuffs, firsthand vegetables and fruits have a short span of maintaining fresh quality, in which there is almost no spoilage. Whereas items like volatile liquids, radioactive chemicals, trendy goods and electronic goods have more span of marinating their quality or freshness. Wu et al. (2006) and Ouyang et al. (2006) were first to incorporate this phenomenon in to the inventory system. They termed it as "non-instantaneous deterioration." In addition, they suggested that if the retailer can successfully lessen the rate of deterioration of an item by enhancing facilities at the storage space, the total annual inventory cost would be lowered. Many researchers (Dye (2013), Jaggi and Verma (2010), Ouyang et al. (2008), Shah et al. (2013), Soni and Patel (2012), Wang et al. (2015), Wu et. al. (2009)) have discussed non-instantaneous deterioration in their study.

For the first time, Hill (1995) formulated an inventory model with ramp-type demand rate. In case of ramp-type demand rate, the rate of demand increases linearly at the beginning; then, it goes constant until the end of replenishment cycle. Such a demand pattern is mostly observed in new brand consumer goods, which are likely to be introduced in the market. The demand rate of such products is generally an increasing function of time to some extent, and then it becomes constant. Many researchers have studied inventory models with ramp-type demand. Cheng and Wang (2009) extended this idea from ramp-type demand to trapezoidal-type demand. Cheng et al. (2011) extended the model for deteriorating items by allowing shortages with partial backlogging. Recent articles on trapezoidal-type demand are Shukla and Suthar (2016), Wu et al. (2016), etc. The previously mentioned study encourages inventory practitioners to formulate one-level or two-level trade credit policies to improve overall profitability of complete supply chain. One may refer articles by Abdulkader et al. (2015), Seifert et al. (2013), Singh et al. (2013), Singh and Pandey (2013), and Soni et al. (2010) which cite an up-to-date review and direct towards the scope of further research.

Here, proposed model deals with retailer's inventory system. It is assumed that retailer deals with an item having trapezoidal-type and credit-period-trended demand rate, i.e., demand of an item increases first up to some level and then it stabilizes over a certain period of time. Thereafter, it again decreases due to either a competitive item for consumption or expiration. It also depends upon credit period offered to the customer. Deterioration is assumed to be non-instantaneous. Moreover, shortages are not expected during ordering cycle. By examining the inventory system, guideline to avail optimal solution is exhibited as an algorithm, which helps to choose optimal strategy with aforementioned assumptions. Numerical examples illustrate proposed model and sensitivity analysis is carried out with respect to major parameters to give managerial insights. The remaining article is outlined section-wise. Section 2 deals with assumption and notations under consideration; Section 3 presents mathematical formulation of an inventory system. In support of this formulation, numerical examples are presented in Section 4, along with special cases. Sensitivity analysis is presented and managerial insights are discussed in Section 5. The learning is concluded in Section 6.

Assumptions and Notations

The following assumptions and notations are used in the formulation of mathematical form of the proposed model.

1. The inventory system deals with a single item. Rate of replenishment rate is assumed to be infinite and lead-time is considered to be zero or negligible. The length of planning horizon is infinite. Inventory system does not possess shortages.
2. The function $I(t)$ represents level of an inventory at any instant of time t during $[0,T]$, where T is length of ordering cycle.
3. To boost demand of an item, retailer offers a credit period M to the buyers. The demand is assumed to be trapezoidal type and depending upon credit period, say $R(t,M)$, defined as under:

$$R(t,M)=\begin{cases}(a_1+b_1t)M^{\beta} & ; \quad 0\le t\le \lambda_1\\ (a_1+b_1\lambda_1)M^{\beta} & ; \quad \lambda_1\le t\le \lambda_2\\ (a_2-b_2t)M^{\beta} & ; \quad \lambda_2\le t\le T\end{cases} \tag{1}$$

   where $a_1>0, a_2>0, b_1>0$ and $b_2>0$ are scale parameters of demand. Demand of an item increases during $[0,\lambda_1]$; thereafter, it remains stable during $[\lambda_1,\lambda_2]$. After time $t=\lambda_2$, it decreases. $\beta>0$ is credit period elasticity.
4. It is observed that longer the credit period offered to the buyer, increases default risk to the retailer. Rate of default risk due to credit period offered is assumed to be $F(M)=1-M^{-\delta}$, where $\delta>0$ is scaling parameter.
5. During the ordering cycle, t_d is the time up to which the product has no deterioration. Thereafter, the product deteriorates with a constant rate 'θ'; $0<\theta<1$. Moreover, it is assumed that deteriorated product is neither repaired, nor replenished during the ordering cycle.
6. Economic order quantity Q (a decision variable) is an initial level of inventory system.
7. We consider P is the selling price/unit, C is the purchase cost/unit, h is the holding cost/unit/time unit and A is the ordering cost/order.

8. $K(M,T)$ is assumed to be average profit of the retailer.

Formulation of Model

Mathematical formulation for an inventory system is developed from retailer's stance. As deterioration is non-instantaneous, the retailer's inventory level depletes due to demand only at initial stage, during $[0,t_d]$; thereafter, it depletes due to demand and deterioration both. λ_1 is the time up to which demand of an item increases; thereafter, up to $t=\lambda_2$ demand remains constant and then demand rate decreases with time.

Depending upon the length of t_d, λ_1 and λ_2, we have three cases: Case 1 $t_d \le \lambda_1$, Case 2 $\lambda_1 < t_d < \lambda_2$ and Case 3 $\lambda_2 \le t_d$.

Case 1: $t_d \le \lambda_1$

The differential equation governing inventory system can be written as follows:

$$\frac{dI(t)}{dt}=\begin{cases}-(a_1+b_1t)M^\beta & ; \quad 0\le t\le t_d\\ -\theta I(t)-(a_1+b_1t)M^\beta & ; \quad t_d\le t\le \lambda_1\\ -\theta I(t)-(a_1+b_1\lambda_1)M^\beta & ; \quad \lambda_1\le t\le\lambda_2\\ -\theta I(t)-(a_2-b_2t)M^\beta & ; \quad \lambda_2\le t\le T\end{cases} \tag{2}$$

with

$$I(T)=0 \tag{3}$$

Using boundary condition (3), continuity of $I(t)$ at time $t=t_d$, $t=\lambda_1$ and $t=\lambda_2$, we solve Equation (2) and solution is:

$$I(t)=\begin{cases}I_{1A}(t) & ; \quad 0\le t\le t_d\\ I_{1B}(t) & ; \quad t_d\le t\le\lambda_1\\ I_{1C}(t) & ; \quad \lambda_1\le t\le\lambda_2\\ I_{1D}(t) & ; \quad \lambda_2\le t\le T\end{cases} \tag{4}$$

where,

$$I_{1A}(t)=M^\beta(t_d-t)\left(\frac{b_1}{2}(t_d+t)+a_1\right)+\frac{M^\beta}{\theta^2}\begin{pmatrix}-Tb_2\theta e^{\theta(T-t)}-b_1e^{\theta(\lambda_1-t)}\\ +e^{\theta(T-t_d)}(a_2\theta+b_2)-(b_1t_d+a_1)\theta\\ +e^{\theta(\lambda_2-t_d)}((b_1\lambda_1+b_2\lambda_2+a_1-a_2)\theta-b_2)+b_1\end{pmatrix}$$

$$I_{1B}(t)=\frac{M^\beta}{\theta^2}\begin{pmatrix}e^{\theta(T-t)}(b_2+a_2\theta-Tb_2\theta)\\ +e^{\theta(\lambda_2-t)}(a_1\theta-a_2\theta+b_1\lambda_1\theta+b_2\lambda_2\theta-b_2)\\ -b_1e^{\theta(\lambda_1-t)}-(b_1t\theta-b_1+a_1\theta)\end{pmatrix}$$

$$I_{1C}(t)=\frac{M^\beta}{\theta^2}(e^{\theta(T-t)}(a_2\theta+b_2-Tb_2\theta)+e^{\theta(\lambda_2-t)}\theta(b_2(\lambda_2-a_2)+b_1\lambda_1+a_1)-\theta(b_1\lambda_1+a_1))$$

$$I_{1D}(t)=\frac{M^\beta}{\theta^2}(e^{\theta(T-t)}(b_2+a_2\theta-Tb_2\theta)-(b_2+a_2\theta-b_2t\theta))$$

Using (4), retailer's initial stock level is Q,

$$Q=I(0)=\frac{M^\beta}{\theta^2}\begin{pmatrix}e^{\theta(T-t_d)}\left((a_2-Tb_2)\theta+b_2\right)+e^{\theta(\lambda_2-t_d)}\\ \left((b_1\lambda_1+b_2\lambda_2+a_1-a_2)\theta-b_2\right)\\ -(b_1t_d+a_1)\theta-b_1\left(e^{\theta(\lambda_1-t_d)}-1\right)\\ +\frac{\theta^2t_d}{2}(t_db_1+2a_1)\end{pmatrix} \tag{5}$$

Total profit of an inventory system comprises following cost components:

1. Net sales revenue after default risk (SR):

$$SR=P\cdot(1-F(M))\begin{pmatrix}\int_0^{\lambda_1}(a_1+b_1t)M^\beta dt\\ +\int_{\lambda_1}^{\lambda_2}(a_1+b_1\lambda_1)M^\beta dt\\ +\int_{\lambda_2}^{T}(a_2-b_2t)M^\beta dt\end{pmatrix} \tag{6}$$

2. Ordering cost (OC):

$$OC=A \tag{7}$$

3. Purchase cost (PC):

$$PC=C\cdot Q=\frac{CM^\beta}{\theta^2}\begin{pmatrix}e^{\theta(T-t_d)}\left((a_2-Tb_2)\theta+b_2\right)+e^{\theta(\lambda_2-t_d)}\\ \left((b_1\lambda_1+b_2\lambda_2+a_1-a_2)\theta-b_2\right)\\ -(b_1t_d+a_1)\theta-b_1\left(e^{\theta(\lambda_1-t_d)}-1\right)\\ +\frac{\theta^2t_d}{2}(t_db_1+2a_1)\end{pmatrix} \tag{8}$$

4. Inventory holding cost (HC):

$$HC = h\int_0^T I(t)dt = h\begin{pmatrix}\int_0^{t_d} I_{1A}(t)dt \\ +\int_{t_d}^{\lambda_1} I_{1B}(t)dt + \int_{\lambda_1}^{\lambda_2} I_{1C}(t)dt + \int_{\lambda_2}^{T} I_{1D}(t)dt\end{pmatrix} \quad (9)$$

5. Cost due to deterioration (DC):

$$DC = D\left(Q - \begin{pmatrix}\int_0^{\lambda_1}(a_1+b_1t)M^\beta dt \\ +\int_{\lambda_1}^{\lambda_2}(a_1+b_1\lambda_1)M^\beta dt \\ +\int_{\lambda_2}^{T}(a_2-b_2t)M^\beta dt\end{pmatrix}\right) \quad (10)$$

So, from Equations (6), (7), (8), (9) and (10), retailer's yearly profit per time unit is,

$$K_1(T,M) = \frac{1}{T}[SR - OC - PC - HC - DC] \quad (11)$$

Case 2: $\lambda_1 < t_d < \lambda_2$

In this case, the differential equation governing inventory system can be written as follows:

$$\frac{dI(t)}{dt} = \begin{cases} -(a_1+b_1t)M^\beta & ; \quad 0 \le t \le \lambda_1 \\ -(a_1+b_1\lambda_1)M^\beta & ; \quad \lambda_1 \le t \le t_d \\ -\theta I(t) - (a_1+b_1\lambda_1)M^\beta & ; \quad t_d \le t \le \lambda_2 \\ -\theta I(t) - (a_2-b_2t)M^\beta & ; \quad \lambda_2 \le t \le T \end{cases} \quad (12)$$

with

$$I(T) = 0 \quad (13)$$

Using boundary condition (13), continuity of $I(t)$ at time $t=\lambda_1$, $t=t_d$ and $t=\lambda_2$, we solve Equation (12) and solution is:

$$I(t) = \begin{cases} I_{2A}(t) & ; \quad 0 \le t \le \lambda_1 \\ I_{2B}(t) & ; \quad \lambda_1 \le t \le t_d \\ I_{2C}(t) & ; \quad t_d \le t \le \lambda_2 \\ I_{2D}(t) & ; \quad \lambda_2 \le t \le T \end{cases} \quad (14)$$

where

$$I_{2A}(t) = \frac{M^\beta}{\theta^2}\begin{pmatrix} e^{\theta(T-t_d)}(-Tb_2\theta + a_2\theta + b_1) \\ +e^{\theta(\lambda_2-t_d)}(b_1\lambda_1\theta + b_2\lambda_2 + a_1\theta - a_2\theta - b_2) \\ +(2t_db_1\lambda_1 + 2t_da_1 - t^2b_1 - 2a_1t - \lambda_1^2b_1) \\ \frac{\theta^2}{2} - (b_1\lambda_1 + a_1)\theta \end{pmatrix}$$

$$I_{2B}(t) = \frac{M^\beta}{\theta^2}\begin{pmatrix} e^{\theta(T-t_d)}(a_2\theta - Tb_2\theta + b_2) \\ +e^{\theta(\lambda_2-t_d)}(b_1\lambda_1\theta + b_2\lambda_2\theta + a_1\theta - a_2\theta - b_2) \\ -b_1\lambda_1t\theta^2 - a_1t\theta^2 - b_1\lambda_1\theta - a_1\theta + t_db_1\lambda_1\theta^2 \\ +t_da_1\theta^2 \end{pmatrix}$$

$$I_{2C}(t) = \frac{M^\beta}{\theta^2}\begin{pmatrix} e^{\theta(T-t)}(-Tb_2\theta + a_2\theta + b_2) + e^{\theta(\lambda_2-t)} \\ (b_2\lambda_2\theta - a_2\theta - b_2 + b_1\lambda_1\theta + a_1\theta) \end{pmatrix} - (b_1\lambda_1 + a_1)\theta$$

$$I_{2D}(t) = \frac{M^\beta}{\theta^2}\begin{pmatrix} e^{\theta(T-t)}(b_2 + a_2\theta - Tb_2\theta) \\ -(b_2 + a_2\theta - b_2t\theta) \end{pmatrix}$$

Using Equation (14), retailer's initial stock level is Q,

$$Q = I(0) = \frac{M^\beta}{\theta^2}\begin{pmatrix} e^{\theta(T-t_d)}(-Tb_2\theta + a_2\theta + b_1) \\ +e^{\theta(\lambda_2-t_d)}(b_1\lambda_1\theta + b_2\lambda_2 + a_1\theta - a_2\theta - b_2) \\ +(2t_db_1\lambda_1 + 2t_da_1 - \lambda_1^2b_1)\frac{\theta^2}{2} \\ -(b_1\lambda_1 + a_1)\theta \end{pmatrix} \quad (15)$$

Total profit of an inventory system comprises following cost components:

Net sales revenue after default risk (SR):

$$SR = P\cdot(1-F(M))\begin{pmatrix}\int_0^{\lambda_1}(a_1+b_1t)M^\beta dt \\ +\int_{\lambda_1}^{\lambda_2}(a_1+b_1\lambda_1)M^\beta dt \\ +\int_{\lambda_2}^{T}(a_2-b_2t)M^\beta dt\end{pmatrix} \quad (16)$$

Ordering Cost (OC):

$$OC = A \quad (17)$$

Purchase cost (PC):

$$PC = C\cdot Q = \frac{CM^\beta}{\theta^2}\begin{pmatrix} e^{\theta(T-t_d)}(-Tb_2\theta + a_2\theta + b_1) \\ +e^{\theta(\lambda_2-t_d)}(b_1\lambda_1\theta + b_2\lambda_2 + a_1\theta - a_2\theta - b_2) \\ +(2t_db_1\lambda_1 + 2t_da_1 - \lambda_1^2b_1)\frac{\theta^2}{2} \\ -(b_1\lambda_1 + a_1)\theta \end{pmatrix} \quad (18)$$

Inventory holding cost (HC):

$$HC = h\int_0^T I(t)\,dt = h\begin{pmatrix} \int_0^{\lambda_1} I_{2A}(t)\,dt + \int_{\lambda_1}^{t_d} I_{2B}(t)\,dt \\ +\int_{t_d}^{\lambda_2} I_{2C}(t)\,dt + \int_{\lambda_2}^{T} I_{2D}(t)\,dt \end{pmatrix} \qquad (19)$$

Cost due to deterioration (DC):

$$DC = D\left(Q - \begin{pmatrix} \int_0^{\lambda_1}(a_1+b_1t)M^\beta dt \\ +\int_{\lambda_1}^{\lambda_2}(a_1+b_1\lambda_1)M^\beta dt \\ +\int_{\lambda_2}^{T}(a_2-b_2t)M^\beta dt \end{pmatrix}\right) \qquad (20)$$

So, from Equations (16), (17), (18), (19) and (20), retailer's yearly profit per time unit is,

$$K_2(T,M) = \frac{1}{T}\left[SR - OC - PC - HC - DC\right] \qquad (21)$$

Case 3: $\lambda_2 \le t_d$

The differential equation governing inventory system for this case can be written as follows:

$$\frac{dI(t)}{dt} = \begin{cases} -(a_1+b_1t)M^\beta & ; \quad 0 \le t \le \lambda_1 \\ -(a_1+b_1\lambda_1)M^\beta & ; \quad \lambda_1 \le t \le \lambda_2 \\ -(a_2-b_2t)M^\beta & ; \quad \lambda_2 \le t \le t_d \\ -\theta I(t) - (a_2-b_2t)M^\beta & ; \quad t_d \le t \le T \end{cases} \qquad (22)$$

with

$$I(T) = 0 \qquad (23)$$

Using boundary condition (23), continuity of $I(t)$ at time $t=\lambda_1$, $t=\lambda_2$ and $t=t_d$ we solve Equation (22) and solution is:

$$I(t) = \begin{cases} I_{3A}(t) & ; \quad 0 \le t \le \lambda_1 \\ I_{3B}(t) & ; \quad \lambda_1 \le t \le \lambda_2 \\ I_{3C}(t) & ; \quad \lambda_2 \le t \le t_d \\ I_{3D}(t) & ; \quad t_d \le t \le T \end{cases} \qquad (24)$$

where

$$I_{3A}(t) = M^\beta\frac{(\lambda_1 - t)}{2}\left(b_1(\lambda_1+t)+2a_1\right) - \frac{M^\beta}{2\theta^2}\begin{pmatrix} 2b_1\theta^2\lambda_1(\lambda_1-\lambda_2) + (t_d^2-\lambda_2^2)b_2\theta^2 \\ +2a_1\theta^2(\lambda_1-\lambda_2) + 2a_2\theta^2(\lambda_2 - t_d) \\ +2(Tb_2\theta - a_2\theta - b_2)e^{\theta(T-t_d)} \\ +2(a_2\theta + b_2 - b_2t_d\theta) \end{pmatrix}$$

$$I_{3B}(t) = M^\beta(a_1+b_1\lambda_1)(\lambda_2 - t) - \frac{M^\beta}{\theta^2}\begin{pmatrix} \left(\frac{(t_d-\lambda_2)\theta^2}{2}\right)\left((t_d+\lambda_2)b_2 - 2a_2\right) \\ +e^{\theta(T-t_d)}(b_2T\theta - a_2\theta - b_2) + (b_2 + a_2\theta - b_2t_d\theta) \end{pmatrix}$$

$$I_{3C}(t) = \frac{M^\beta}{2}\begin{pmatrix} -t^2b_1 - 2a_1t - \lambda_1^2b_1 + 2b_1\lambda_1\lambda_2 + \lambda_2^2b_2 \\ -t_d^2b_2 + 2a_1\lambda_2 - 2a_2\lambda_2 + 2a_2t_d \\ +\frac{2(a_2\theta + b_2 - b_2T\theta)e^{\theta(T-t_d)} - 2(a_2\theta + b_2 - b_2t_d\theta)}{\theta^2} \end{pmatrix}$$

$$I_{3D}(t) = \frac{M^\beta}{\theta^2}\left(e^{\theta(T-t)}(b_2 + a_2\theta - Tb_2\theta) - (b_2 + a_2\theta - b_2t\theta)\right)$$

Using Equation (24), retailer's initial stock level is Q,

$$Q = I(0) = M^\beta\frac{\lambda_1}{2}(b_1\lambda_1 + 2a_1) - \frac{M^\beta}{2\theta^2}\begin{pmatrix} 2b_1\theta^2\lambda_1(\lambda_1-\lambda_2) \\ +(t_d^2-\lambda_2^2)b_2\theta^2 + 2a_1\theta^2(\lambda_1-\lambda_2) \\ +2a_2\theta^2(\lambda_2 - t_d) \\ +2(Tb_2\theta - a_2\theta - b_2)e^{\theta(T-t_d)} \\ +2(a_2\theta + b_2 - b_2t_d\theta) \end{pmatrix} \qquad (25)$$

Total profit of an inventory system comprises following cost components:

Net sales revenue after default risk (SR):

$$SR = P\cdot(1-F(M))\begin{pmatrix} \int_0^{\lambda_1}(a_1+b_1t)M^\beta dt \\ +\int_{\lambda_1}^{\lambda_2}(a_1+b_1\lambda_1)M^\beta dt \\ +\int_{\lambda_2}^{T}(a_2-b_2t)M^\beta dt \end{pmatrix} \qquad (26)$$

Ordering Cost (OC):

$$OC = A \qquad (27)$$

Purchase cost (PC):

$$PC = C\cdot Q = CM^\beta\frac{\lambda_1}{2}(b_1\lambda_1 + 2a_1) - \frac{CM^\beta}{2\theta^2}\begin{pmatrix} 2b_1\theta^2\lambda_1(\lambda_1-\lambda_2) + (t_d^2-\lambda_2^2)b_2\theta^2 \\ +2a_1\theta^2(\lambda_1-\lambda_2) + 2a_2\theta^2(\lambda_2 - t_d) \\ +2(Tb_2\theta - a_2\theta - b_2)e^{\theta(T-t_d)} \\ +2(a_2\theta + b_2 - b_2t_d\theta) \end{pmatrix} \qquad (28)$$

Inventory holding cost (HC):

$$HC = h\int_0^T I(t)dt = h\left(\begin{array}{l}\int_0^{\lambda_1} I_{3A}(t)dt + \int_{\lambda_1}^{\lambda_2} I_{3B}(t)dt \\ +\int_{\lambda_2}^{t_d} I_{3C}(t)dt + \int_{t_d}^{T} I_{3D}(t)dt\end{array}\right) \quad (29)$$

Cost due to deterioration (DC):

$$DC = D\left(Q - \left(\begin{array}{l}\int_0^{\lambda_1}(a_1+b_1t)M^{\beta}dt \\ +\int_{\lambda_1}^{\lambda_2}(a_1+b_1\lambda_1)M^{\beta}dt \\ +\int_{\lambda_2}^{T}(a_2-b_2t)M^{\beta}dt\end{array}\right)\right) \quad (30)$$

So, from Equations (26), (27), (28), (29) and (30), retailer's yearly profit per time unit is,

$$K_3(T,M) = \frac{1}{T}\left[SR - OC - PC - HC - DC\right] \quad (31)$$

Using Equations (11), (21) and (31), retailer's yearly profit per time unit is defined as under:

$$K(T,M) = \begin{cases} K_1(T,M) & ; \quad 0 \le t_d \le \lambda_1 \\ K_2(T,M) & ; \quad \lambda_1 < t_d < \lambda_2 \\ K_3(T,M) & ; \quad \lambda_2 \le t_d \le T \end{cases}$$

To evaluate optimal solution for $K(T,M)$, we use fundamental calculus and computational algorithm given as under:

Computational Algorithm

Step 1 Assign parametric values to the parameter.

Step 2 If $t_d \le \lambda_1$, then go to Step 3,

Else, if $\lambda_1 < t_d < \lambda_2$ then go to Step 4, otherwise go to Step 5.

Step 3 Solve $\frac{\partial K_1}{\partial M} = 0$ and $\frac{\partial K_1}{\partial T} = 0$ to find optimal values M^* and T^* for which $\frac{\partial^2 K_1}{\partial M^2} < 0$ and $\frac{\partial^2 K_1}{\partial M^2}\frac{\partial^2 K_1}{\partial T^2} - \left(\frac{\partial^2 K_1}{\partial M \partial T}\right)^2 > 0$ then go to Step 6.

Step 4 Solve $\frac{\partial K_2}{\partial M} = 0$ and $\frac{\partial K_2}{\partial T} = 0$ to find optimal values M^* and T^* for which $\frac{\partial^2 K_2}{\partial M^2} < 0$ and $\frac{\partial^2 K_2}{\partial M^2}\frac{\partial^2 K_2}{\partial T^2} - \left(\frac{\partial^2 K_2}{\partial M \partial T}\right)^2 > 0$ then go to Step 6.

Step 5 Solve $\frac{\partial K_3}{\partial M} = 0$ and $\frac{\partial K_3}{\partial T} = 0$ to find optimal values M^* and T^* for which $\frac{\partial^2 K_3}{\partial M^2} < 0$ and $\frac{\partial^2 K_3}{\partial M^2}\frac{\partial^2 K_3}{\partial T^2} - \left(\frac{\partial^2 K_3}{\partial M \partial T}\right)^2 > 0$ then go to Step 6.

Step 6 Using M^* and T^* find EOQ Q and maximum value of $K(T^*, M^*)$.

Numerical Example

To illustrate previous formulation and computational algorithm, we consider following examples.

Example 1 For Case 1: Consider standard values to the parameters $A = \$100$/order, $C = \$10$/unit, $h = \$2$/unit/time unit, $D = \$2$/deteriorated unit, $P = \$12$/unit, $\lambda_1 = 100/365$ years, $\lambda_2 = 200/365$ years, $t_d = 50/365$ years, $a_1 = 1000$, $b_1 = 0.2$, $a_2 = 1000$, $b_2 = 0.1$, $\delta = 2$, $\theta = 0.5$. With this set of parameters, optimal credit period and cycle time are $M^* = 0.7536$ years and $T^* = 0.2952$ years, respectively. Using these optimal values EOQ $Q = 97.30$ units and maximized total profit is $K = \$3069.15$. The graph given in "Fig. 2" indicates that the total profit per time unit is strictly concave. Variation of profit with respect to T and M is exhibited too.

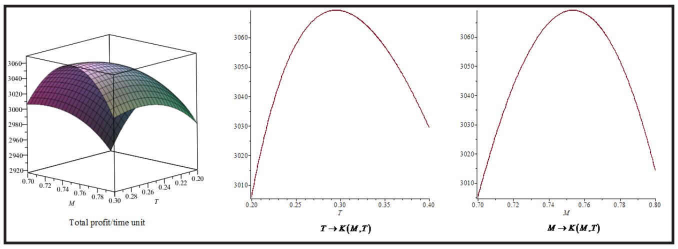

Fig. 1: Concavity of Profit function (For Example 1)

Example 2 For Case 2: Consider standard values to the parameters $A = \$100$/order, $C = \$10$/unit, $h = \$2$/unit/time unit, $D = \$2$/deteriorated unit, $P = \$12$/unit, $\lambda_1 = 100/365$ years, $\lambda_2 = 200/365$ years, $t_d = 150/365$ years, $a_1 = 1000$, $b_1 = 0.2$, $a_2 = 1000$, $b_2 = 0.1$, $\delta = 2$, $\theta = 0.5$ With this set of parameters, optimal credit period and cycle time are $M^* = 0.6922$ years and $T^* = 0.4898$ years, respectively. Using these optimal values EOQ $Q = 112.64$ units and maximized total profit is $K = \$2670.29$. The graph given in "Fig. 2" indicates that the total profit per time unit is strictly concave. Variation of profit with respect to T and M is exhibited too.

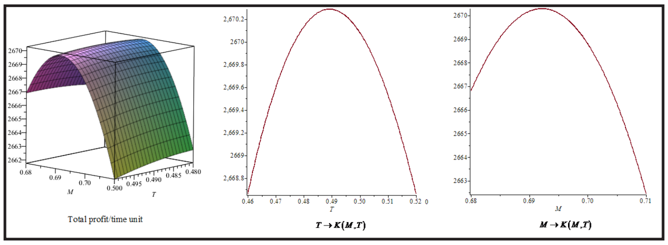

Fig. 2: Concavity of Profit function (For Example 2)

Example 3 For Case 3: Consider standard values to the parameters $A = \$100$/order, $C = \$10$/unit, $h = \$2$/unit/time unit, $D = \$2$/deteriorated unit, $P = \$12$/unit, $\lambda_1 = 100/365$ years, $\lambda_2 = 200/365$ years, $t_d = 250/365$ years, $a_1 = 1000$, $b_1 = 0.2$, $a_2 = 1000$, $b_2 = 0.1$, $\delta = 2$, $\theta = 0.5$. With this set of parameters, optimal credit period and cycle time are $M^* = 0.7502$ years and $T^* = 0.6584$ years respectively. Using these optimal values EOQ $Q = 208.55$ units and maximized total profit is $K = \$3224.69$. The graph given in "Fig. 3" indicates that the total profit per time unit is strictly concave. Variation of profit with respect to T and M is exhibited too.

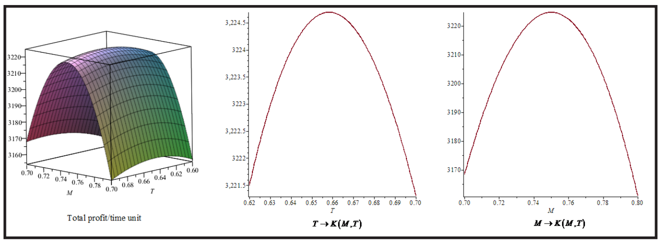

Fig. 3: Concavity of Profit function (For Example 3)

Remark: To replace trapezoidal type demand with ramp-type demand, one may take $\lambda_2 \to T$ and by considering $t_d \to 0$ case with instantaneous deterioration can be derived. $\theta \to 0$ presents an inventory system without deterioration. To discuss such special cases and illustrate them, we consider following examples.

Example 4: Consider $\lambda_2 \to T$ in (1), $t_d \to 0$ and $\theta \to 0$ in (2) (or (12) or (22)), presents an inventory system for items having ramp-type demand without deterioration. Consider standard values to the parameters $A = \$100$/order, $C = \$10$/unit, $h = \$2$/unit/time unit, $P = \$12$/unit, $\lambda = 100/365$ years, $a = 1000$, $b = 0.5$, $\delta = 2$. With this set of parameters, optimal credit period and cycle time are $M^* = 0.7539$ years and $T^* = 0.5565$ years, respectively. Using these optimal values EOQ $Q = 179.79$ units and maximized total profit is $K = \$3230.87$. The graph given in "Fig. 4" indicates that the total profit per time unit is strictly concave.

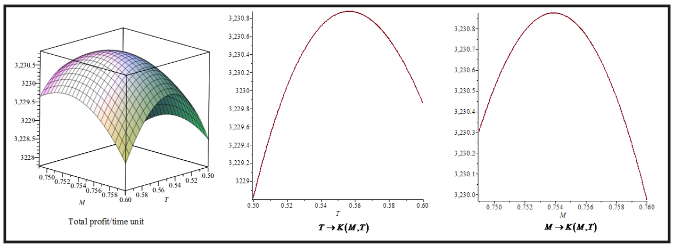

Fig. 4: Concavity of Profit function (For Example 4)

Example 5: Consider $\lambda_2 \to T$ in (1), $t_d \to 0$ in (2) (or (12) or (22)), presents an inventory system for items having ramp-type demand with instantaneous deterioration. Consider standard values to the parameters $A = \$100$/order, $C = \$10$/unit, $h = \$2$/unit/time unit, $P = \$12$/unit, $D = \$2$/deteriorated unit, $\theta = 0.4$, $\lambda = 100/365$ years, $a = 1000$, $b = 0.5$, $\delta = 2$. With this set of parameters, optimal credit period and cycle time are $M^* = 0.7359$ years and $T^* = 0.3040$ years, respectively. Using these optimal values EOQ $Q = 94.87$ units and maximized total profit is $K = \$2921.33$. The graph given in "Fig. 5" indicates that the total profit per time unit is strictly concave.

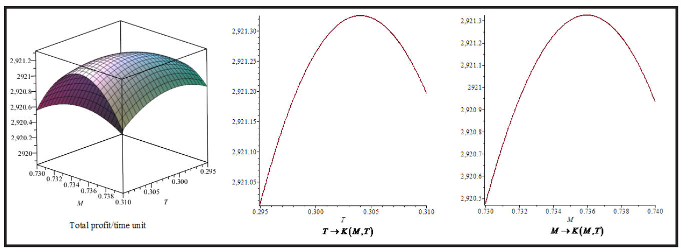

Fig. 5: Concavity of Profit function (For Example 5)

Example 6: Consider $t_d \to 0$ and $\theta \to 0$ in (2) (or (12) or (22)), presents an inventory system for items having trapezoidal-type demand without deterioration. Consider standard values to the parameters $A = \$100$/order, $C = \$10$/unit, $h = \$2$/unit/time unit, $P = \$12$/unit, $\lambda_1 = 100/365$ years, $\lambda_2 = 200/365$ years, $a_1 = 1000$, $b_1 = 0.2$, $a_2 = 1000$, $b_2 = 0.1$, $\delta = 2$. With this set of parameters, optimal credit period and cycle time are $M^* = 0.7539$ years and $T^* = 0.5559$ years, respectively. Using these optimal values EOQ $Q = 179.62$ units and maximized total profit is $K = \$3230.59$. The graph given in "Fig. 6" indicates that the total profit per time unit is strictly concave.

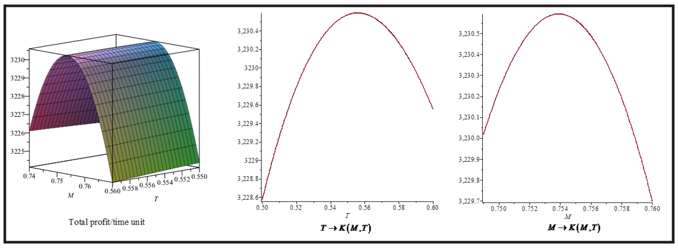

Fig. 6: Concavity of Profit function (For Example 6)

Example 7: Consider $t_d \to 0$ and $\theta \to 0$ in (2) (or (12) or (22)), presents an inventory system for items having trapezoidal type demand with instantaneous deterioration. Consider standard values to the parameters $A = \$1000$/order, $C = \$10$/unit, $h = \$2$/unit/time unit, $D = \$2$/deteriorated unit, $P = \$12$/unit, $\lambda_1 = 100/365$ years, $\lambda_2 = 200/365$ years, $a_1 = 1000$, $b_1 = 0.2$, $a_2 = 1000$, $b_2 = 0.1$, $\delta = 2$, $\theta = 0.5$, With this set of parameters, optimal credit period and cycle time are $M^* = 0.5593$ years and $T^* = 1.2249$ years, respectively. Using these optimal values EOQ $Q = 165.38$ units and maximized total profit is $K = \$1060.61$. The graph given in "Fig. 7" indicates that the total profit per time unit is strictly concave.

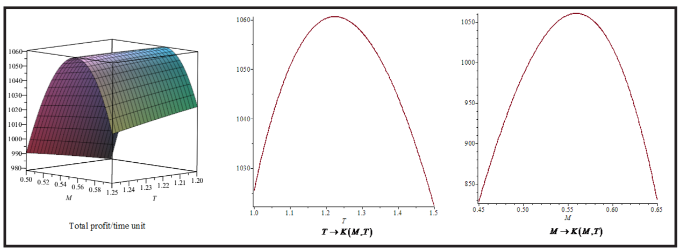

Fig. 7: Concavity of Profit function (For Example 7)

Example 8: Consider $\lambda_2 \to T$ in (1), presents an inventory system for items having ramp-type demand with non-instantaneous deterioration. Consider standard values to the parameters $A = \$100$/order, $C = \$10$/unit, $h = \$2$/unit/time unit, $P = \$12$/unit, $D = \$2$/deteriorated unit, $\theta = 0.4$, $\lambda = 100/365$ years, $t_d = 50/365$ years, $a = 1000$, $b = 0.5$, $\delta = 2$. With this set of parameters, optimal credit period and cycle time are $M^* = 0.7585$ years and $T^* = 0.3635$ years, respectively. Using these optimal values EOQ $Q = 121.02$ units and maximized total profit is $K = \$3177.28$. The graph given in "Fig. 8" indicates that the total profit per time unit is strictly concave.

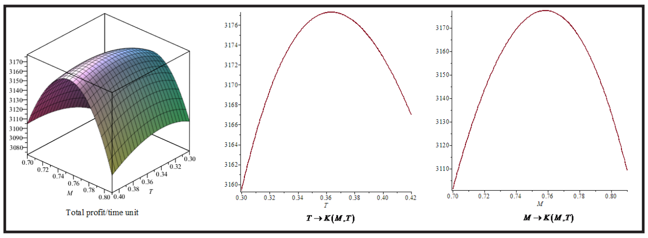

Fig. 8: Concavity of Profit function (For Example 8)

Sensitivity Analysis

Variations in optimal values, EQO and total profit per time unit with respect to variation in major parameters, like $a_1, a_2, C, P, h, D, \beta, \delta, \theta$ and A are exhibited in Figs 9-12.

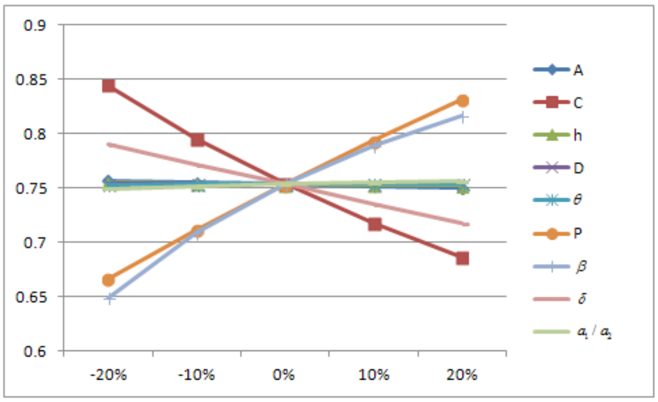

Fig. 9: Variation in M

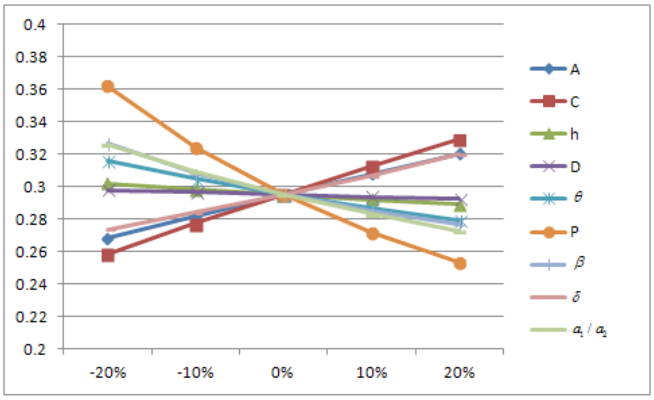

Fig. 10: Variation in T

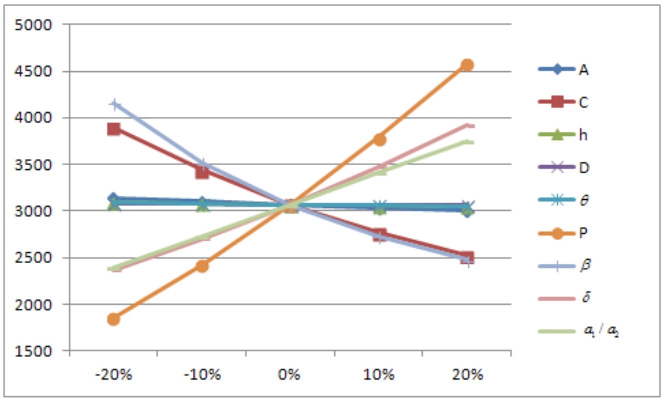

Fig. 11: Variation in $K(M,T)$

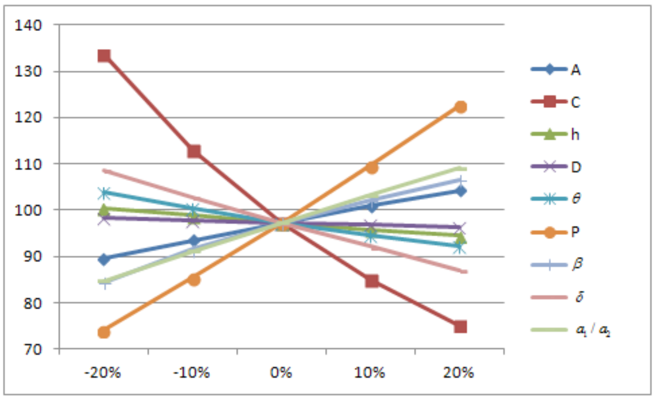

Fig. 12: Variation in Q

Managerial Insights

Using sensitivity analysis, the following managerial issues were observed.

From "Fig. 9", it is observed that credit period M is highly sensitive with respect to C, P, β and δ. It has positive impact on respect to selling price and credit period elasticity. Also, it has opposite impact on purchase price, holding cost, ordering cost.

From "Fig. 10", it is observed that cycle time T is highly sensitive with respect to purchase cost, selling price and ordering cost. It has positive impact on increase in ordering cost and purchase cost. In addition, it has opposite impact on increase in holding cost, deterioration, rate of deterioration, selling price and credit period elasticity.

From "Fig. 11", it is observed that the total profit of an inventory system K is highly sensitive with respect to selling price. In addition, it has opposite impact on increase in ordering cost, purchase cost, holding cost, deterioration cost, rate of deterioration and credit period elasticity.

From "Fig. 12", it is observed that economic order quantity Q is highly sensitive with respect to selling price and purchase cost. It has positive impact on increase in ordering cost, credit period elasticity and selling price. In addition, it has opposite impact over increase in purchase cost, deterioration cost, holding cost and rate of deterioration.

Increase in scaling parameter of default risk gives positive impact on length of ordering cycle and total profit of an inventory system. Moreover, it has opposite impact on length of credit period and economic order quantity.

Conclusion

It is difficult for the retailer to decide permissible credit period to be offered to the customer. Increase in the credit period offered increases the default risks. An optimal ordering and credit period policy for the item having trapezoidal type demand is presented here from the point of retailer. The non-instantaneous deterioration is observed over units in the inventory system. During cycle time, shortages are not allowed. A solution procedure is discussed with support of various numerical examples. Computational algorithm is illustrated through examples and managerial insights were presented using sensitivity analysis.

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