

# Service Halt in Repairable Redundant Machining System with Degraded Failure and Set-up

Neetu Singh

World College of Technology and Management, Gurugram, Haryana, India.  
Email: [neetusingh1608@gmail.com](mailto:neetusingh1608@gmail.com)

**Abstract:** In the present paper we study single server machining system having operating as well as mixed standby units. The service station is subject to breakdown while in operation. The life times and repair times of the failed units and service station are assumed to be exponentially distributed. The set up time for the repairing of service station is also exponentially distributed. In case when all standby units are exhausted, the system may also operate in degraded mode with less than M operating units, which are required for normal functioning of the system. We employ recursive method to obtain the steady state probability distribution of the number of failed units in the system. The performance indices such as expected number of failed units in the system, average waiting time, throughput, etc. are obtained in closed form in terms of probabilities.

**Keywords:** Degraded failure, Machine repair, Mixed standby, Queue size, Recursive method, Set up.

## I. INTRODUCTION

In modern age of computer systems and automated manufacturing systems, the provision of providing spares is common so that the multi-unit systems may operate continuously over a long run without interruption. The reliability of the system may be improved if we provide as many spare units as possible for each module of the system. However, due to some constraints (e.g. volume, cost, space, etc.), it is not feasible in real time systems. Sometimes the combination of cold and warm standbys is used to overcome this difficulty. The standbys may be of three types, cold, warm and hot. Failure rate of cold standby is zero, whereas the failure rate of warm standby unit is less than that of an operating unit. For hot standby the failure rate is same as that of an operating unit. Several researchers have studied the concept of providing spares in machining system. A profit model in machine repair problem for warm standby system was studied by Sivazlian and Wang [3]. Jain and Premlata [21] developed the M/M/R machine repair model with reneging and spares. Gupta and

Rao [28] developed the M/G/1 machine interference model with spares. Jain [19] examined (m, M) machine repair model with spares and state dependent rate. Wang and Kuo [14] gave cost and probabilistic analysis of mixed standby components. Performance modeling of machine system with mixed standby components implementing the balking and reneging behavior of costumers was studied by Sharma *et al.* [7]. Recently, modelling of machining system with standbys have been done by several researchers including Jain *et al.* [24], Singh *et al.* [26], Kuo and Ke [5]. Shekhar *et al.* [30] analyzed repairable redundant system with switching failure and geometric reneging. Singh and Rani [31] have done a short survey on recent developments in mathematical analysis of queues for machine repair problem.

An important stream of study for machining system includes server breakdown. When the service station breaks down, it is subject to repair if there is a maintenance facility available. As soon as a machine fails it is attended by the server till any server is available, otherwise it will queue up and wait for repairing. The concept of breakdown was considered for queueing models in different frame-works from time to time by many researchers. Shogan [2] examined a single server queue with arrival rate dependent on server breakdown. Profit analysis of the M/M/R machine repair problem with spares and server breakdown was discussed by Wang [13]. Optimal N-policy for single server Markovian queue with breakdown, repair and state dependent arrival rate was discussed by Jain [20]. Cost analysis of the M/M/R machine repair problem with balking, reneging and server breakdowns was considered by Ke and Wang [11]. Grey *et al.* [29] investigated a vacation queueing model with service breakdowns. Cost analysis of finite M/M/R queueing system with balking, reneging and server breakdowns was given by Wang and Chang [15]. Management policy for M/M/1 queue with server breakdowns and vacations was discussed by Ke and Pearn [12]. Later, Choudhury *et al.* [9] and Singh *et al.* [6] have investigated queueing system with server breakdown.

Degraded failure mode occurs when some units in a multi component system fail but the system is still able to execute its function in short mode due to paucity of operating units which are required for the normal functioning of the machining system. When all spare units are being used and a unit fails,

then the system is said to be a short system as it works under stress and causes degradation in operating units. Somani *et al.* [1] developed the reliability model with latent failures using Markov chains. Hokstad and Frovig [27] made a degraded and critical failure model for a standby component with dormant failures. They derived the so-called naked failure rate. They also obtained the safety unavailability of the dormant system. Jain *et al.* [22] gave reliability analysis of gracefully degradable multi processors system. Machine repair problem with spares and degraded failure was investigated by Jain *et al.* [23]. Tran and Bukkapatnam [10] considered modeling and analysis of the coupled dynamics of machine degradation and repair processes using piecewise affine stochastic differential equations.

When the server breaks down it is subject to repair to restore its functioning. The repairman takes set up time before it starts repairing of service station. Queueing models with set up time have drawn the attention of research workers who are interested in the application of queueing theory in production and manufacturing processes. (cf. Huan *et al.* [18]; Choudhury [8]; Reddy *et al.* [17]; Jain *et al.* [25]; Wang *et al.* [16]; Krishna Kumar *et al.* [4]).

In the present paper we attempt to obtain queue size distribution for a multi components machining system having warm and cold standby units, when an operating unit fails, it is sent for repair, if server is available otherwise waits in the queue for its turn for repair. An available spare unit replaces the operating unit on failure. In the case when all spares are being used and the operating unit fails the system works in degraded mode due to shared load. The remaining paper is organized as follows. The mathematical formulation and underlined assumptions of the model are given in Section 2. Section 3 provides the steady state queue size distribution, which is obtained by applying the recursive method. Various performance characteristics of the system are calculated in Section 4. Section 5 provides some special cases. Conclusions are drawn in the last Section 6.

## II. THE MODEL DESCRIPTION

We consider a machine repair problem having M operating units, mixed standbys, and service interruption due to breakdown. The following assumptions are made to formulate the problem mathematically:

- The system consists of M-operating, S warm standby and Y cold standby units. For the normal functioning of the system, M operating units are required. However, system can operate in short mode when there are less than M but more than p operating units.
- The lifetime distributions of operating unit, warm standby units are exponentially distributed with rate,  $\lambda$  and  $\gamma$ , respectively.
- When all standby units are being used then the operating unit fails with degraded failure rate  $\lambda_{n-\overline{Y+S}}$  as there is degradation in operating units due to stress.

- If at any time an operating unit fails, it is sent for repair, if repairman is available; otherwise waits in queue to be repaired by single repairman according to first come first served discipline. An available cold spare unit replaces the operating unit on failure. However, if all cold spares are already used, then hot spare replaces the failed unit.
- The single repairman restores failed unit exponentially with rate  $\mu$  when some or all cold standby spares are available. The repairman switches to faster rate  $\mu_1$  when all cold standbys are being used; in case when all warm standbys are also used, he repairs with rate  $\mu_2 (> \mu_1)$ .
- If the repairing of a failed unit is completed, it becomes as good as new after repair and goes to the operating group if there are less than M operating units; otherwise it joins the standby group. Whenever a spare unit is taken into operation, its characteristics are same as that of already operating units.
- The service station may breakdown while in operation. Life time of service station is exponentially distributed with rate  $\alpha$ , set up rate of repairman is  $\nu$  and the repair time of service station is exponentially distributed with rate  $\beta$ .
- The system may be in any of the following mutually exclusive states:
  - (i) State (0, n), ( $0 \leq n \leq K$ ) indicates the state having n failed units in the system when service station is in working state, its probability is denoted by P (0, n).
  - (ii) State (1, n), ( $0 \leq n \leq K$ ) indicates the state having n failed units in the system when service station is in breakdown state, its probability is denoted by P (1, n).
  - (iii) State (2, n), ( $0 \leq n \leq K$ ) indicates the state having n failed units in the system when service station is in repair state, its probability is denoted by P (2, n).
- The state dependent failure rates can be expressed as:

$$\lambda_n = \begin{cases} M\lambda + S\gamma & : (0 \leq n \leq Y) \\ M\lambda + (S + Y - n)\gamma & : (Y + 1 \leq n \leq Y + S) \\ (M + Y + S - n)\lambda_{n-\overline{Y+S}} & : (Y + S + 1 \leq n \leq K) \end{cases}$$

where,  $K = M + S + Y - p$

The state dependent service rates are:

$$\mu_n = \begin{cases} \mu_0 & : (0 \leq n \leq Y) \\ \mu_1 & : (Y + 1 \leq n \leq Y + S) \\ \mu_2 & : (Y + S + 1 \leq n \leq K) \end{cases}$$

## III. THE STEADY STATE EQUATIONS AND ANALYSIS

Using state dependent failure and repair rates, the steady state difference equations governing the model are as follows:

$$(M\lambda + \lambda_C + S\gamma + \alpha)P_{0,0} = \beta P_{2,0} + \mu_0 P_{0,1} \quad \dots(1)$$

$$(M\lambda + \lambda_C + S\gamma + \alpha + \mu_0)P_{0,n} = (M\lambda + S\gamma)P_{0,n-1} + \beta P_{2,n} \quad 1 \leq n \leq Y-1 \quad \dots(2)$$

$$(M\lambda + \lambda_C + S\gamma + \alpha + \mu_0)P_{0,Y} = (M\lambda + S\gamma)P_{0,Y-1} + \beta P_{2,Y} + \mu_1 P_{0,Y+1} \quad \dots(3)$$

$$[M\lambda + \lambda_C + (S+Y-n)\gamma + \alpha + \mu_1]P_{0,n} = [M\lambda + (S+Y-n+1)\gamma]P_{0,n-1} + \beta P_{2,n} + \mu_1 P_{0,n+1}, \quad \dots(4)$$

$$[M\lambda + \lambda_C + \alpha + \mu_1]P_{0,Y+S} = [M\lambda + \gamma]P_{0,Y+S-1} + \beta P_{2,Y+S} + \mu_2 P_{0,Y+S+1} \quad \dots(5)$$

$$[(M+Y+S-n)\lambda_{n-\overline{Y+S}} + \lambda_C + \alpha + \mu_2]P_{0,n} = [(M+Y+S-n+1)\lambda_{n-1-\overline{Y+S}}]P_{0,n-1} + \beta P_{2,n} + \mu_2 P_{2,n+1}, Y+S+1 \leq n \leq K-1 \quad \dots(6)$$

$$(\alpha + \mu_2)P_{0,K} = \lambda_{M-m+1}P_{0,K-1} + \beta P_{2,K} \quad \dots(7)$$

$$(M\lambda + S\gamma + v)P_{1,0} = \alpha P_{0,0} \quad \dots(8)$$

$$(M\lambda + S\gamma + v)P_{1,n} = (M\lambda + S\gamma)P_{1,n-1} + \alpha P_{0,n}, 1 \leq n \leq Y \quad \dots(9)$$

$$[M\lambda + (S+Y-n)\gamma + v]P_{1,n} = [M\lambda + (S+Y-n+1)\gamma]P_{1,n-1} + \alpha P_{0,n}, Y+1 \leq n \leq Y+S \quad \dots(10)$$

$$(M\lambda + v)P_{1,Y+S} = (M\lambda + \gamma)P_{1,Y+S-1} + \alpha P_{0,Y+S} \quad \dots(11)$$

$$[(M+Y+S-n)\lambda_{n-\overline{Y+S}} + v]P_{1,n} = [(M+Y+S-n+1)\lambda_{n-1-\overline{Y+S}}]P_{1,n-1} + \alpha P_{0,n}, Y+S+1 \leq n \leq K-1 \quad \dots(12)$$

$$vP_{1,K} = \lambda_{M-m+1}P_{1,K-1} + \alpha P_{0,K} \quad \dots(13)$$

$$[M\lambda + S\gamma + \beta]P_{2,0} = vP_{1,0} \quad \dots(14)$$

$$[M\lambda + S\gamma + \beta]P_{2,n} = [M\lambda + S\gamma]P_{2,n-1} + vP_{1,n}, 1 \leq n \leq Y \quad \dots(15)$$

$$[M\lambda + (S+Y-n)\gamma + \beta]P_{2,n} = [M\lambda + (S+Y-n)\gamma]P_{2,n-1} + vP_{1,n}, Y+1 \leq n \leq Y+S \quad \dots(16)$$

$$[M\lambda + \beta]P_{2,Y+S} = [M\lambda + \gamma]P_{2,Y+S+1} + vP_{1,Y+S} \quad \dots(17)$$

$$[(M+Y+S-n)\lambda_{n-\overline{Y+S}} + \beta]P_{2,n} = [(M+Y+S-n+1)\lambda_{n-1-\overline{Y+S}}]P_{2,n-1} + vP_{1,n}, Y+S+1 \leq n \leq K-1 \quad \dots(18)$$

$$\beta P_{2,K} = \lambda_{M-m+1}P_{2,K-1} + vP_{1,K} \quad \dots(19)$$

We employ recursive method to obtain the solution of Eqs. (1)-(19) by taking

$$P_{m,n} = q_{m,n}P_{0,0}, m=0, 1, 2 \text{ and } 0 \leq n \leq K \quad \dots(20)$$

The normalizing condition  $\sum_{m=0}^2 \sum_{n=0}^K P_{m,n} = 1$  is used to determine  $P_{0,0}$  as:

$$P_{0,0} \left[ \sum_{m=0}^2 \sum_{n=0}^K q_{m,n} \right] \quad \dots(21)$$

To obtain  $q_{m,0}$ , we use Eqs. (20), (8), (14) and get

$$q_{m,0} = \begin{cases} 1 & , m = 0 \\ \varphi_0^{(1)} & , m = 1 \\ \varphi_0^{(2)} \varphi_0^{(1)} & , m = 2 \end{cases} \quad \dots(22)$$

Now we obtain recursive relation for  $q_{m,n}$  for different intervals as follows:

A. For  $1 \leq n \leq Y$

We obtain the value of  $q_{m,n}$  by using Eqs. (1), (9), (15), (2) respectively:

$$q_{0,n} = \varphi_0^{(3)} - \frac{\beta}{\mu_0} \varphi_0^{(2)} \varphi_0^{(1)}, n = 1 \quad \dots(23)$$

$$q_{m,n} = \begin{cases} \frac{\varphi_0}{\varphi_0 + v} q_{1,n-1} + \varphi_0^{(1)} q_{0,n}, m = 1 \\ \frac{\varphi_0}{\varphi_0 + \beta} q_{2,n-1} + \varphi_0^{(2)} q_{1,n}, m = 2 \end{cases} \quad \dots(24)$$

and

$$q_{0,n} = (1 + \varphi_0^{(3)})q_{0,n-1} - \frac{\varphi_0}{\mu_0} q_{0,n-2} - \frac{\beta}{\mu_0} q_{2,n-1}, 2 \leq n \leq Y \quad \dots(25)$$

where,

$$\varphi_n = M\lambda + (S-n)\gamma, 0 \leq n \leq Y; \varphi_n^{(1)} = \frac{\alpha}{\varphi_n + v}, 0 \leq n \leq Y$$

$$\varphi_n^{(2)} = \frac{v}{\varphi_n + \beta}, 0 \leq n \leq Y; \varphi_n^{(3)} = \frac{\varphi_n + \alpha}{\mu_0}, 0 \leq n \leq Y-1$$

B. For  $Y+1 \leq n \leq Y+S$

On solving Eqs. (3), (10), (16) and (4) respectively, we get

$$q_{0,n} = (\theta_n^{(3)} + \frac{\mu_0}{\mu_1})q_{0,Y} - \frac{\varphi_0}{\mu_1} q_{0,Y-1} - \frac{\beta}{\mu_1} q_{2,Y}, n = Y+1 \quad \dots(26)$$

$$q_{m,n} = \begin{cases} \frac{\theta_{n-1}}{\theta_n + v} q_{1,n-1} + \theta_n^{(1)} q_{0,n}, m = 1 \\ \frac{\theta_{n-1}}{\theta_n + \beta} q_{2,n-1} + \theta_n^{(2)} q_{1,n}, m = 2 \end{cases} \quad \dots(27)$$

$$q_{0,n} = (1 + \theta_{n-1}^{(3)})q_{0,n-1} - \frac{\theta_{n-2}}{\mu_1}q_{0,n-2} - \frac{\beta}{\mu_1}q_{2,n-1},$$

$$Y + 2 \leq n \leq Y + S \quad \dots(28)$$

where,

$$\theta_n = M\lambda + (S + Y - n)\gamma, \quad Y + 1 \leq n \leq Y + S,$$

$$\theta_n^{(1)} = \frac{\alpha}{\theta_n + \nu}, \quad Y + 1 \leq n \leq Y + S,$$

$$\theta_n^{(2)} = \frac{\nu}{\theta_n + \beta}, \quad Y + 1 \leq n \leq Y + S,$$

$$\theta_n^{(3)} = \frac{\theta_n + \alpha}{\mu_1}, \quad Y + 1 \leq n \leq Y + S - 1$$

C. For  $Y + S + 1 \leq n \leq K$

The solution of Eqs. (5), (12), (18), (6) is given by:

$$q_{0,n} = (\psi'_{Y+S} + \frac{\mu_1}{\mu_2})q_{0,Y+S} - \frac{\theta_{Y+S-1}}{\mu_2}q_{0,Y+S-1} - \frac{\beta}{\mu_2}q_{2,Y+S}, \quad n = Y + S + 1 \quad \dots(29)$$

$$q_{m,n} = \begin{cases} \frac{\psi_{n-1}}{\psi_n + \nu}q_{1,n-1} + \psi_n^{(1)}q_{0,n}, & m = 1 \\ \frac{\psi_{n-1}}{\psi_n + \beta}q_{2,n-1} + \psi_n^{(2)}q_{1,n}, & m = 2 \end{cases} \quad \dots(30)$$

$$q_{0,n} = (1 + \psi_{n-1}^{(3)})q_{0,n-1} - \frac{\psi_{n-2}}{\mu_2}q_{0,n-2} - \frac{\beta}{\mu_2}q_{2,n-1},$$

$$Y + S + 2 \leq n \leq K \quad \dots(31)$$

Let  $P_n$  be the probability that there are n-failed units in the service station, then we have

$$P_n = P_{0,n} + P_{1,n} + P_{2,n}, \quad 0 \leq n \leq K \quad \dots(32)$$

Substituting the value of  $P_{m,n}$  from equation (20) and using

$$P_n = g_n P_{0,0}, \quad 0 \leq n \leq K \quad \dots(33)$$

we get

$$P_{0,0} = \left[ \sum_{n=0}^K g_n \right]^{-1}$$

Eqs. (32) and (33), give

$$g_n = q_{0,n} + q_{1,n} + q_{2,n} \quad \dots(34)$$

On solving Eqs. (22)-(31), we get  $g_n$  as:

$$g_0 = q_{0,0} + q_{1,0} + q_{2,0} = 1 + \varphi_0^{(1)} + \varphi_0 \varphi_0^{(1)} \quad \dots(35)$$

$$g_1 = \varphi_0^{(3)} + \varphi_0^{(1)} \varphi_0^{(2)} \left[ \left\{ \frac{\varphi_0}{\varphi_0 + \beta} - \frac{\beta}{\mu_0} \right\} + \varphi_0^{(1)} \left( 1 + \varphi_0^{(2)} \right) \left\{ \varphi_0^{(3)} - \frac{\beta}{\mu_0} \varphi_0^{(2)} \varphi_0^{(1)} + \frac{\varphi_0}{\varphi_0 + \nu} \right\} \right] \quad \dots(36)$$

$$g_n = \left[ 1 + \varphi_0^{(1)} (1 + \varphi_0^{(2)}) \right] \left[ (1 + \varphi_0^{(3)})q_{0,n-1} - \frac{\varphi_0}{\mu_0}q_{0,n-2} \right] + (1 + \varphi_0^{(2)}) \left( \frac{\varphi_0}{\varphi_0 + \nu} \right) q_{1,n-1} + \left[ \frac{\varphi_0}{\varphi_0 + \beta} - \frac{\beta}{\mu_0} \right] \left[ 1 + \varphi_0^{(1)} (1 + \varphi_0^{(2)}) \right] q_{2,n-1}, \quad 1 \leq n \leq Y \quad \dots(37)$$

$$g_{Y+1} = \left[ 1 + \theta_{Y+1}^{(1)} (1 + \theta_{Y+1}^{(2)}) \right] \left[ \left( \theta_Y^{(3)} + \frac{\mu_0}{\mu_1} \right) q_{0,Y} - \frac{\varphi_0}{\mu_1} q_{0,Y-1} \right] + (1 + \theta_{Y+1}^{(2)}) \left( \frac{\theta_Y}{\theta_{Y+1} + \nu} \right) q_{1,Y} + \left[ \frac{\theta_Y}{\theta_{Y+1} + \beta} - \frac{\beta}{\mu_1} \left\{ 1 + \theta_{Y+1}^{(1)} (1 + \theta_{Y+1}^{(2)}) \right\} \right] q_{2,Y} \quad \dots(38)$$

$$g_n = \left[ 1 + \theta_n^{(1)} (1 + \theta_n^{(2)}) \right] \left[ (1 + \theta_{n-1}^{(3)})q_{0,n-1} - \frac{\theta_{n-2}}{\mu_1}q_{0,n-2} \right] + (1 + \theta_n^{(2)}) \left( \frac{\theta_{n-1}}{\theta_n + \nu} \right) q_{1,n-1} + \left[ \frac{\theta_{n-1}}{\theta_n + \beta} - \frac{\beta}{\mu_1} \left\{ 1 + \theta_n^{(1)} (1 + \theta_n^{(2)}) \right\} \right] q_{2,n-1}, \quad Y + 2 \leq n \leq Y + S \quad \dots(39)$$

$$g_{Y+S+1} = \left[ 1 + \psi_{Y+S+1}^{(1)} \left( 1 + \psi_{Y+S+1}^{(2)} \right) \right] \left[ \left\{ \psi'_{Y+S} + \frac{\mu_1}{\mu_2} \right\} q_{0,Y+S} - \frac{\theta_{Y+S-1}}{\mu_2} q_{0,Y+S} \right] + \left( 1 + \psi_{Y+S+1}^{(2)} \right) \frac{\psi_{Y+S}}{\psi_{Y+S+1} + \nu} q_{1,Y+S} + \left[ \frac{\psi_{Y+S}}{\psi_{Y+S+1}} - \frac{\beta}{\mu_2} \left\{ 1 + \psi_{Y+S+1}^{(1)} \left( 1 + \psi_{Y+S+1}^{(2)} \right) \right\} \right] q_{2,Y+S} \quad \dots(40)$$

$$\begin{aligned}
 g_n = & \left\{ 1 + \psi_n^{(1)} \left( 1 + \psi_n^{(2)} \right) \right\} \left[ \left\{ 1 + \psi_{n-1}^{(3)} \right\} q_{0,n-1} - \frac{\psi_{n-2}}{\mu_2} q_{0,n-2} \right] \\
 & + \left( 1 + \psi_n^{(2)} \right) \left( \frac{\psi_{n-1}}{\psi_n + \nu} \right) q_{1,n-1} \\
 & + \left[ \frac{\psi_{n-1}}{\psi_n + \beta} - \frac{\beta}{\mu_2} \left\{ 1 + \psi_n^{(1)} \left( 1 + \psi_n^{(2)} \right) \right\} \right] \\
 & q_{2,n-1}, Y + S + 2 \leq n \leq K
 \end{aligned} \quad \dots(41)$$

#### IV. SOME PERFORMANCE MEASURES

In this section, we establish the expressions for various measures of performance characterizing the system in terms of steady state probabilities obtained in previous section.

The expected number of failed units in the queue is:

$$E(N_q) = \left[ \sum_{n=1}^K n q_{0,n} + \sum_{m=1}^2 \sum_{n=1}^K (n-1) q_{m,n} \right] P_{0,0} \quad \dots(42)$$

The expected number of failed units in the system is:

$$E(N_S) = \sum_{n=0}^K n g_n P_{0,0} \quad \dots(43)$$

The expected waiting time in the queue is:

$$W_q = \frac{E(N_q)}{\lambda_{eff}} \quad \dots(44)$$

where,

$$\begin{aligned}
 \lambda_{eff} = & \sum_{n=0}^Y (M\lambda + S\gamma) g_n P_{0,0} + \sum_{n=Y+1}^{Y+S} \{ M\lambda + (S+Y-n)\gamma \} \\
 & g_n P_{0,0} + \sum_{n=Y+S+1}^K (M+S+Y-n) \lambda_{n-Y+S} g_n P_{0,0}
 \end{aligned}$$

The expected waiting time in the system is:

$$W_S = \frac{E(N_S)}{\lambda_{eff}} \quad \dots(45)$$

Throughput is obtained as:

$$T = \mu_0 \sum_{n=0}^Y g_n P_{0,0} + \mu_1 \sum_{n=Y+1}^{Y+S} g_n P_{0,0} + \mu_2 \sum_{n=Y+S+1}^K g_n P_{0,0} \quad \dots(46)$$

#### IV. SOME SPECIAL CASES

By setting appropriate parameters we can deduce special cases as follows:

##### A. Case-I: Machine Repair Problem with Spares, Degraded Failure and Server Breakdown

When  $\nu=0$ , we get the results for multi-components machining system with mixed standbys, degraded failure and server breakdown. In particular when we use only warm spares (i.e.  $Y=0$ ) the results are obtained for machine repair problem with warm standbys, degraded failure and server breakdown as discussed by Jain and Singh (2005).

##### B. Case-II: Machine Repair Problem with Spares, Interruption and Set Up

If  $\lambda_{n-Y+S} = \lambda = \text{constant}$ , then our model reduces for machining system with mixed standbys, interruption and set up. Also, if we take  $Y=0$ , results for machine repair problem with warm standbys, interruption and set up are obtained.

##### C. Case-III: Machine Repair Problem with Spares

When  $\lambda_{n-Y+S} = \lambda = \text{constant}$ ,  $\alpha=0$ , results for machine repair problem with mixed standbys are obtained. Further, to deduce results for machining system with warm spares we substitute  $Y=0$ .

## VI. CONCLUSION

We have investigated a multi component machining system with mixed standbys. The single service station, which renders service, is subject to random breakdown and repair. The steady state queue size distribution is obtained by applying the recursive method. The provision of spares may be helpful to facilitate the uninterrupted service. Our model can play a crucial role in computer, communication and manufacturing systems. The incorporation of degraded failure and set up time makes our model closer to real life situations

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