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e-open Sets on Intuitionistic Fuzzy Topological Spaces in Šostak's Sense

G. Saravanakumar[1], S. Tamilselvan[2] and A. Vadivel[3*]
[1]Department of Mathematics, M.Kumarasamy College of Engineering, Karur, Tamil Nadu, India.
Email: saravananguru2612@gmail.com
[2]Mathematics Section (FEAT), Annamalai University, Annamalainagar, Tamil Nadu, India.
Email: tamilau@yahoo.com
[3]Department of Mathematics, Government Arts College, Karur, Tamil Nadu, India.
Email: avmaths@gmail.com
*Corresponding Author

Abstract: We introduce the concepts of fuzzy (ι,κ)-e (resp. (ι,κ)-δ pre and (ι,κ)-δ semi) open sets, their respective interior and closure operators on intuitionistic fuzzy topological spaces in Šostak's sense and then we investigate some of their characteristic properties.

Keywords and Phrases: Fuzzy (ι,κ)-eopen set, Fuzzy(ι,κ)-einterior, Fuzzy (ι,κ)-e closure, Intuitionistic fuzzy topology in Šostak's sense.

AMS (2000) Subject Classification: 54A40.

I. Introduction

The concept of fuzzy sets was introduced by Zadeh [13]. Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which developed by Ŝostak [12], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay *et al.* [3] and by Ramadan [10]. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy set (briefly, IFS) was introduced by Atanassov [1]. Recently, Coker and his colleagues [4, 6, 7] introduced intuitionistic fuzzy topological spaces (briefly, IFTS's) using IFS's. Using the idea of degree of openness and degree of nonopenness, Coker and Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense (SoIFTS, for short) as a generalization of smooth fuzzy topological spaces and IFTS's. In this paper, we introduce the concepts of fuzzy (ι,κ)-*e* (resp. (ι,κ)-δ pre and (ι,κ)-δ semi) open sets, their respective interior and closure operators on SoIFTS and then we investigate some of their characteristic properties.

II. Preliminaries

Let I be the unit interval [0,1] of the real line. A member μ of I^X is called a fuzzy set of X. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement of $\tilde{1}-\mu$. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An IFS K is an ordered pair $K = (\mu_K, \gamma_K)$

where the functions $\mu_K : X \to I$ and $\gamma_K : X \to I$ denote the degree of membership and degree of non-membership, respectively, and $\mu_K + \gamma_K \leq \tilde{1}$.

Obviously every fuzzy set μ on X is an IFS of the form $(\mu, \tilde{1}-\mu)$.

Definition 1 [1] Let $K = (\mu_K, \gamma_K)$ and $L = (\mu_L, \gamma_L)$ be IFS's on X. Then

(i) $K \subseteq L$ iff $\mu_K \leq \mu_L$ and $\gamma_K \geq \gamma_L$,
(ii) $K = L$ iff $K \subseteq L$ and $L \subseteq K$,
(iii) $Kc = (\gamma_K, \mu_K)$,
(iv) $K \cap L = (\mu_K \wedge \mu_L, \gamma_K \vee \gamma_L)$,
(v) $K \cup L = (\mu_K \vee \mu_L, \gamma_K \wedge \gamma_L)$,
(vi) $\sim 0 = (\tilde{0}, \tilde{1})$ and
$\sim 1 = (\tilde{1}, \tilde{0})$.

Definition 2 [1] Let f be a map from a set X to a set Y. Let $K = (\mu_K, \gamma_K)$ be a IFS of X and $L = (\mu_L, \gamma_L)$ an IFS of Y. Then:

(i) The image of K under f, denoted by $f(K)$ is an IFS in Y defined by
$f(K) = (f(\mu K), \tilde{1} - f(\tilde{1} - \gamma K))$.
(ii) The inverse image of L under f, denoted by − is an IFS in X defined by
$f^{-1}(L) = (f^{-1}(\mu_L), f^{-1}(\gamma_L))$.

Definition 3 [10] A smooth fuzzy topology on X is a map → which satisfies the following properties:

(i) $T(\tilde{0}) = T(\tilde{1}) = 1_{\sim}$,
(ii) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$,
(iii) $T(\vee \mu_i) \geq \wedge T(\mu_i)$.

The pair (X,T) is called a smooth fuzzy topological space.

Definition 4 [5] An IFT on X is a family T of IFSs in X which satisfies the following properties:

(i) $0,\sim 1 \in T$,

(ii) If $K_1, K_2 \in T$, then $K_1 \cap K_2 \in T$,

(iii) If $\underline{K}_1 \in T$ for all i, then $\cup K_i \in T$.

The pair (X,T) is called an IFTS.

Let $I(X)$ be a family of all IFS's of X and let $I\otimes I$ be the set of the pair (ι,κ) such that $\iota,k \in I$ and ι+κ£1 .

Definition 5 [6] Let X be a nonempty set. An intuitionistic fuzzy topology in Šostak's sense (SoIFT, for short) $T = (T_1, T_2)$ on X is a map $T{:}I(X) \rightarrow I \otimes I$ which satisfies the following properties:

(i) $T_1(0_{\sim})=T_1(1_{\sim})=1$ and $T_2(0_{\sim})=T_2(1_{\sim})=1$,

(ii) $T_1(K\cap L) \geq T_1(K) \wedge T_1(L)$ and $T_2(K\cap L) \leq T_2(K) \vee T_2(L)$,

(iii) $T_1(\cup K_i) \geq \wedge T_1(K_i)$ and $T_2(\cup K_i) \leq \vee T_2(K_i)$.

The (X, T) = (X, T1, T2) is said to be SoIFTS. Also, we call a gradation of openness of K and a gradation of nonopenness of K.

Definition 6 [8] Let K be an IFS in a SoIFTS(X, T1, T2) and $(\iota,\kappa) \in I\otimes I$. Then K is said to be

(i) fuzzy (ι,κ)-open (briefly, (ι,κ)-fo) if $T_1(K) \geq \iota$ and $\leq \kappa$,

(ii) fuzzy (ι,κ)-closed (briefly, (ι,κ)-fc) if $T_1(K^c) \geq \iota$ and $T_2(K^c) \leq \kappa$.

Definition 7 [8] Let (X, T1, T2) be a SoIFTS. For each $(\iota,\kappa) \in I\otimes I$ and for each $K \in I(X)$, the fuzzy (ι,κ)-interior is defined by

int(*K*,ι,κ)=È{*L*∈*I*(*X*)|*K*⊒*L*,*L is* (ι,κ)-*fo*}.

and the fuzzy (ι,κ)-closure is defined by

cl(*K*,ι,κ)=Ç{*L*∈*I*(*X*)|*K*⊑*L*,*L is* (ι,κ)-*fc*}.

The operators $int{:}I(X)\times I\otimes I \rightarrow I(X)$ and $cl{:}I\times I \otimes I \rightarrow I(X)$ are called the fuzzy interior operator and fuzzy closure operator in (X, T1, T2) respectively.

Lemma 1 [8] For an IFS K in a SoIFTS(X, T1, T2) and $(\iota,\kappa) \in I\otimes I$

(i) $int(K, \iota, \kappa)^c = cl(K^c, \iota, \kappa)$

(ii) $cl(K, \iota, \kappa)^c = int(K^c, \iota, \kappa)$.

Definition 8 [8] Let (X,T) be a SoIFTS. Then it is easy to see that for each $(\iota,\kappa) \in I\otimes I$, the family $T_{(\iota,\kappa)}$ defined by

$$T_{(\iota,\kappa)} = \{K \in I(X) | T_1(K) \geq \iota \text{ and } T_2(K) \leq \kappa\}.$$

is an intuitionistic fuzzy topology on X.

Definition 9 [8] Let (X,T) be an IFTS $(\iota,\kappa) \in I\otimes I$. Then the map $T_{(\iota,\kappa)} : I(X) \rightarrow I \otimes I$ defined by:

$$T^{(\iota,\kappa)}(K) = \begin{cases} (1,0), & \text{if } K = \underset{\sim}{0}, \underset{\sim}{1} \\ (\iota,\kappa), & \text{if } K \in T - \{\underset{\sim}{0}, \underset{\sim}{1}\} \\ (0,1), & \text{otherwise,} \end{cases}$$

becomes an SoIFTS on X.

Definition 10 [11] Let K be an IFS in a SoIFTS(X, T1, T2) and $(\iota,\kappa) \in I\otimes I$. Then K is said to be

(i) fuzzy (ι,κ)-regular open (briefly, (ι,κ)-fro) if $K=int(cl(K,\iota,\kappa),\iota,\kappa)$,

(ii) fuzzy (ι,κ)-regular closed (briefly, (ι,κ)-frc) if $K=cl(int(K,\iota,\kappa),\iota,\kappa)$,

III. Fuzzy (ι, κ)-e (Resp. (ι, κ)-δ Semi and (ι, κ)-δ Pre) open Sets

Definition 1 Let (X, T1, T2) be a SoIFTS. For each $(\iota,\kappa) \in I\otimes I$ and for each $K \in I(X)$, the fuzzy (ι,κ)-δ interior is defined by

$int\delta(K, \iota, \kappa) = \cup\{L \in I(X) | K \supseteq L, L \text{ is } (\iota, \kappa)\text{-fro}\}$.

and the fuzzy (ι,κ)-δ closure is defined by

$cl\delta(K, \iota, \kappa) = \cap\{L \in I(X) | K \subseteq L, L \text{ is } (\iota, \kappa)\text{-frc}\}$.

Definition 2 Let K be an IFS in a SoIFTS(X, T1, T2) and $(\iota,\kappa) \in I\otimes I$. Then K is said to be fuzzy

(i) (ι,κ)-dpre open (briefly, (ι,κ)-fdpo) if $K \subseteq int(cl\delta(K, \iota, \kappa), \iota, \kappa)$,

(ii) (ι,κ)-dpre closed (briefly, (ι,κ)-fdpc) if $K \supseteq cl(int\delta(K, \iota, \kappa), \iota, \kappa)$,

(iii) (ι, κ)-δsemi open (briefly, (ι, κ)-fδso) if $K \subseteq cl(int\delta(K, \iota, \kappa), \iota, \kappa)$,

(iv) (ι, κ)-δsemi closed (briefly, (ι, κ)-fδsc) if $K \supseteq int(cl\delta(K, \iota, \kappa), \iota, \kappa)$,

(v) (ι, κ)-e open (briefly, (ι, κ)-feo) if $K \subseteq int(cl\delta(K, \iota, \kappa), \iota, \kappa) \cup cl(int\delta(K, \iota, \kappa), \iota, \kappa)$,

(vi) (ι, κ)-e closed (briefly, (ι, κ)-fec) if $K \supseteq int(cl\delta(K, \iota, \kappa), \iota, \kappa) \cap cl(int\delta(K, \iota, \kappa), \iota, \kappa)$.

Theorem 1

(i) Every (ι,κ)-fo set is (ι,κ)-fdso set,

(ii) Every (ι,κ)-foset is (ι,κ)-fdpo set,

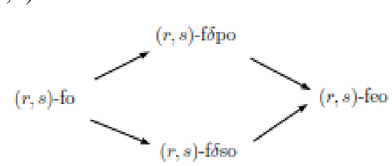
(iii) Every (ι,κ)-fdso set is (ι,κ)-feo set,

(iv) Every (ι,κ)-fdpo set is (ι,κ)-feo set.

Proof. We prove only (i) the others are similar. Let K be a (ι,κ)-fro set, clearly K be (ι,κ)-fo set. Since every (ι,κ)-fro set is (ι,κ)-fo set. Therefore

$A = intcl(A) \subseteq int\delta(A) \Rightarrow cl(A) \subseteq clint\delta(A) \Rightarrow A \subseteq cl(A) \subseteq clint\delta(A) \Rightarrow A \subseteq clint\delta(A)$.

Remark 1 From the Theorem 1 it is clear that the implications are true for $(\iota,\kappa) \in I\otimes I$



The converses of the above implications are not true as the following examples shows:

Example 1 Let *X*={*x*,*y*} and let and be IFS of *X* defined as:

K1(x) = (0.8, 0.1), K1(y) = (0.5, 0.1);

K2(x) = (0.4, 0.3), K2(y) = (0.5, 0.3).

Define T : I(X) → I ⊗ I by

$$T(K) = (T_1(K), T_2(K)) = \begin{cases} (1,0) & \text{if } K = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } K = K_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly (T1, T2) is a SoIFT on *X*. The IFS is (1/2,1/2)-fdpo which is not (1/2,1/2)-fo also which is (1/2,1/2)-f*e*o which is not (1/2,1/2)-fdso.

Example 2 Let *X*={*x*,*y*} and let and be IFS of *X* defined as: K1(x) = (0.1, 0.8), K1(y) = (0.1, 0.5);

K2(x) = (0.2, 0.4), K2(y) = (0.2, 0.4).

Define *T*:*I*(*X*) → *I* ⊗ *I* by:

$$T(K) = (T_1(K), T_2(K)) = \begin{cases} (1,0) & \text{if } K = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } K = K_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly (T1, T2) is a SoIFT on *X*. The IFS is (1/2,1/2)-fδ so which is not (1/2,1/2)-fo also which is (1/2,1/2)-f*e*o which is not (1/2,1/2)-fdpo.

Theorem 2 Let *K* be an IFS in an SoIFTS(X, T1, T2 and (ι,κ)∈*I*⊗*I* then

(i) pclδ(K, ι, κ) ⊇ K ∪ cl(intδ(K, ι, κ), ι, κ) and pintδ(K, ι, κ) ⊆ K ∩ int(clδ(K, ι, κ), ι, κ)

(ii) sclδ(K, ι, κ) ⊇ K ∪ int(clδ(K, ι, κ), ι, κ) and sintδ(K, ι, κ) ⊆ K ∩ cl(intδ(K, ι, κ), ι, κ).

Proof. We will prove only the first statement of (i) and the others is similar. Since pclδ(K, ι, κ) is f(ι,κ)-dpc set, we have cl(intδ(K, ι, κ), ι, κ) ⊆ clintδ(pclδ(K, ι, κ), ι, κ) ⊆ pclδ(K, ι, κ). Thus K ∪ cl(intδ(K, ι, κ), ι, κ) ⊆ pclδ(K, ι, κ).

Theorem 3 Let *K* be an IFS in an SoIFTS(X, T1, T2) and (ι,κ)∈*I*⊗*I*. Then the following statements are equivalent:

(i) *K* is (ι,κ)-f*e*o

(ii) K = pintδ(K, ι, κ) ∪ sintδ(K, ι, κ).

Proof. Let *K* be a (ι,κ)-f*e*o set. Then K ⊆ cl(intδ(K, ι, κ), ι, κ) ∪ int(clδ(K, ι, κ), ι, κ). . By Theorem 2, we have pintδ(K, ι, κ) ∪ sintδ(K, ι, κ) = (K ∩ int(clδ(K, ι, κ), ι, κ)) ∪ (K ∩ cl(intδ(K, ι, κ), ι, κ)) = K ∩ (int(clδ(K, ι, κ), ι, κ) ∪cl(intδ(K, ι, κ), ι, κ)) = K.

Conversely, if K = pintδ(K, ι, κ) ∪ sintδ(K, ι, κ) then, by Theorem 2 K = pintδ(K, ι, κ) ∪ sintδ(K, ι, κ) = (K ∩ int(clδ(K, ι, κ), ι, κ))∪(K∩cl(intδ(K, ι, κ), ι, κ)) = K∩(int(clδ(K, ι, κ), ι, κ)∪cl(intδ(K, ι, κ), ι, κ)) ⊆ int(clδ(K, ι, κ), ι, κ)∪cl(intδ(K, ι, κ), ι, κ) and hence *K* is a (ι,κ)-f*e*o set.

Theorem 4 Let *K* be an IFS in an SoIFTS(X, T1, T2) and (ι,κ)∈*I*⊗*I*. Then the following statements are equivalent:

(i) *K* is (ι,κ)-f*e*c

(ii) K = pintδ(K, ι, κ) ∩ sintδ(K, ι, κ).

Proof. Let *K* be a (ι,κ)-f*e*c set. Then K ⊇ cl(intδ(K, ι, κ), ι, κ) ∩ int(clδ(K, ι, κ), ι, κ).. By Theorem 2, we have pintδ(K, ι, κ) ∩ sintδ(K, ι, κ) = (K ∪ int(clδ(K, ι, κ), ι, κ)) ∩ (K ∪ cl(intδ(K, ι, κ), ι, κ)) = K ∪ (int(clδ(K, ι, κ), ι, κ) ∩cl(intδ(K, ι, κ), ι, κ)) = K.

Conversely, if K = pintδ(K, ι, κ) ∩ sintδ(K, ι, κ) then, by Theorem 2 K = pintδ(K, ι, κ) ∩ sintδ(K, ι, κ) = (K ∪int(clδ(K, ι, κ), ι, κ))∩(K∪cl(intδ(K, ι, κ), ι, κ)) = K∪(int(clδ(K, ι, κ), ι, κ)∩cl(intδ(K, ι, κ), ι, κ)) ⊇ int(clδ(K, ι, κ), ι, κ)∩cl(intδ(K, ι, κ), ι, κ) and hence *K* is a (ι,κ)-f*e*c set.

Theorem 5 Let *K* be an IFS in a SoIFTS(X, T1, T2) and (ι,κ)∈*I*⊗*I*. Then the following statements are equivalent:

(i) *K* is (ι,κ)-fdpo,

(ii) K ⊆ int(clδ(K, ι, κ), ι, κ).

Proof. It follows from Theorem 3.

Theorem 6 Let *K* be an IFS in a SoIFTS(X, T1, T2) and (ι,κ)∈*I*⊗*I*. Then the following statements are equivalent:

(i) *K* is (ι,κ)-fdpc,

(ii) K ⊇ cl(intδ(K, ι, κ), ι, κ).

Proof. It follows from Theorem 4.

Theorem 7 Let *K* be an IFS in a SoIFTS(X, T1, T2) and (ι,κ)∈*I*⊗*I*. Then the following statements are equivalent:

(i) *K* is (ι,κ)-fdso,

(ii) K ⊆ cl(intδ(K, ι, κ), ι, κ).

Proof. It follows from Theorem 3.

Theorem 8 Let *K* be an IFS in a SoIFTS(X, T1, T2) and (ι,κ)∈*I*⊗*I*. Then the following statements are equivalent:

(i) *K* is (ι,κ)-fdsc,

(ii) K ⊇ int(clδ(K, ι, κ), ι, κ).

Proof. It follows from Theorem 4.

Theorem 9 Let (X, T1, T2) be a SoIFTS and (ι,κ)∈*I*⊗*I*

(i) If is a family of (ι,κ)-f*e* (resp. (ι,κ)-fds and (ι,κ)-fdp) - open sets of *X*, then is fuzzy (ι,κ)-f*e* (resp. (ι,κ)-fds and (ι,κ)-fdp) - open,

(ii) If is a family of (ι,κ)-f*e* (resp. (ι,κ)-fds and (ι,κ)-fdp) - closed sets of *X*, then is fuzzy (ι,κ)-f*e* (resp. (ι,κ)-fds and (ι,κ)-fdp) - closed,

Proof. (i) Let {Ki, i ∈ I} be a collection of (ι,κ)-f*e*o sets. Then Ki ⊆ cl(intδ(Ki, ι, κ), ι, κ) ∪ int(clδ(Ki, ι, κ), ι, κ),, hence ∪i Ki ⊆ ∪i(cl(intδ(Ki, ι, κ), ι, κ) ∪ int(clδ(Ki, ι, κ), ι, κ)) ⊆ cl(intδ(∪i Ki, ι, κ), ι, κ) ∪ int(clδ(∪i Ki, ι, κ), ι, κ) for all i ∈ I. for all *i*∈*I*. Then ∪i Ki is (ι, κ)-f*e*o.

(ii) Similar to (i).

The other cases are similar as in (i) and (ii).

Definition 3 Let (X, T1, T2) be a SoIFTS. For each (ι,κ)∈*I*⊗*I* and for each *K*∈*I*(*X*), the (ι,κ)-f*e* (resp. (ι,κ)-fdp and (ι,κ)-fds)-interior is defined by:

eint(*K*,ι,κ) (*resp*. d*pint*(*K*,ι,κ) *and* d*sint*(*K*,ι,κ))={*L*∈*I*(*X*)|*K*⊒*L*, *L is* (ι,κ)-*fe* (*resp*. *f*δ *p and f*δ *s*) *-open*}.

and the (ι,κ)-f*e* (resp. (ι,κ)-fdp and (ι,κ)-fds) - closure is defined by:

esp. d*pcl*(*K*,ι,κ) *and* d*scl*(*K*,ι,κ))={*L*∈*I*(*X*)|*K*⊑*L*, *L isfuzzy* (ι,κ)-*fe* (*resp*. *f*δ *pre and f*δ *semi*) *-closed*}. Obviously *ecl*(*K*,ι,κ) (resp. d*pcl*(*K*,ι,κ) *and* d*scl*(*K*,ι,κ)) is the smallest (ι,κ)-f*e* (resp. fdp and fds) - closed set which contains *K* and *eint*(*K*,ι,κ) (resp. d*pcl*(*K*,ι,κ) *and* d*scl*(*K*,ι,κ)) is the greatest (ι,κ)-f*e* (resp. fdp and fds) - open set which is contained in *K*. Also, *ecl*(*K*,ι,κ) (*resp*. d*pcl*(*K*,ι,κ) *and* d*scl*(*K*,ι,κ))=*K* for any (ι,κ)-f*e* (resp. fdp and fδ s) - closed set *K* and *eint*(*K*,ι,κ) (*resp*. d*pint*(*K*,ι,κ) *and* d*sint*(*K*,ι,κ))=*K* for any (ι,κ)-f*e* (resp. fdp and fds) - open set *K*. Moreover, we have

int(*K*,ι,κ)Í*eint*(*K*,ι,κ)⊑*K*Í*ecl*(*K*,ι,κ)Í*cl*(*K*,ι,κ),

int(*K*,ι,κ)Íd*pint*(*K*,ι,κ)⊑*K*Íd*pcl*(*K*,ι,κ)Í*cl*(*K*,ι,κ),

int(*K*,ι,κ)Íd*sint*(*K*,ι,κ)⊑*K*Íd*scl*(*K*,ι,κ)Í*cl*(*K*,ι,κ).

Also, we have the following results:

(i) ecl(0~, ι, κ) = ~0, ecl(i) ecl(0~, ι, κ) = ~0, ecl(1~, ι, κ) = ~1,

(ii) ecl(K, ι, κ) ⊇ K,

(iii) ecl(K ∪ L, ι, κ) ⊇ ecl(K, ι, κ) ∪ ecl(L, ι, κ)

(iv) ecl(rscl(K, ι, κ), ι, κ) = ecl(K, ι, κ)

(v) eint(0~, ι, κ) = ~0, eint(1~, ι, κ) = ~1,

(vi) eint(K, ι, κ) ⊆ K,

(vii) eint(K ∩ L, ι, κ) ⊆ eint(K, ι, κ) ∩ eint(L, ι, κ)

(viii) eint(eint(K, ι, κ), ι, κ) = eint(K, ι, κ)1~, ι, κ) = ~1,

Theorem 10 Let be SoIFTS. For ∈ and (ι,κ)∈*I*⊗*I*. It satisfies the following statements:

(i) K is (ι, κ)-feo ⇔ K = eint(K, ι, κ),

(ii) K is (ι, κ)-fec ⇔ K = ecl(K, ι, κ),

(iii) ecl(0~, ι, κ) = ~0,

(iv) int(K, ι, κ) ⊆ eint(K, ι, κ) ⊆ K ⊆ ecl(K, ι, κ) ⊆ cl(K, ι, κ),

(v) ecl(K, ι, κ) ∪ ecl(L, ι, κ) ⊆ ecl(K ∪ L, ι, κ),

(vi) cl(ecl(K, ι, κ), ι, κ) = ecl(cl(K, ι, κ), ι, κ) = cl(K, ι, κ).

Proof. (i) Let K be (ι, κ)-feo. Then

eint(K, ι, κ) = ∪{G ∈ IX : G ⊆ K, G is (ι, κ) feo}= K

Conversely, let K = eint(K, ι, κ). Since eint(K, ι, κ) is the arbitrary union of (ι, κ)-feo, then K is (ι, κ)-feo.

(ii) It is similar to part (i).

(iii) It is easily obtained from Definition 2.

(iv) Since int(K, ι, κ) = ∪{G ∈ IX : G ⊆ K, G is (ι, κ) - fo}

⊆ ∪{G ∈ IX : G ⊆ K, G is (ι, κ) -feo}

= eint(K, ι, κ).

It follows that, int(K, ι, κ) ⊆ eint(K, ι, κ). Also,

ecl(K, ι, κ) = ∩{G ∈ IX : G ⊇ K, G is (ι, κ) -fec}⊆ cl(K, ι, κ).

Finally, we have int(K, ι, κ) ⊆ eint(K, ι, κ) ⊆ K ⊆ ecl(K, ι, κ) ⊆ cl(K, ι, κ).

(v) Since, L ⊆ L ∨ G, G ⊆ L ∨ G. Then

ecl(L, ι, κ) ⊆ ecl(L ∪ G, ι, κ) and ecl(G, ι, κ) ⊆ ecl(L ∪ G, ι, κ).

Hence, ecl(L, ι, κ) ∪ ecl(G, ι, κ) ⊆ ecl(L ∪ G, ι, κ).

(vi) Since cl(K, ι, κ) is (ι, κ)-fec set, then ecl(cl(K, ι, κ), ι, κ) = cl(K, ι, κ). (1)

Now it remains to prove only the relation:cl(ecl(K, ι, κ), ι, κ) = cl(K, ι, κ).

Since, K ⊆ ecl(K, ι, κ), then cl(K, ι, κ) ⊆ cl(ecl(K, ι, κ)). It remains to prove: cl(ecl(K, ι, κ), ι, κ) ⊆ cl(K, ι, κ). Let the contrary, that is, cl(ecl(K, ι, κ), ι, κ) ⊆cl(K, ι, κ). Then cl(ecl(K, ι, κ), ι, κ) ⊃ cl(K, ι, κ). So, there exists (ι, κ)-fc setG ∈ IX, G ⊇ K such thatcl(K, ι, κ)(x) ⊂ G(x) ⊂ cl(ecl(K, ι, κ), ι, κ)(x). (2)

Since, K ⊆ G ⇒ ecl(K, ι, κ) ⊆ ecl(G, ι, κ) = ecl(cl(K, ι, κ), ι, κ) = cl(G, ι, κ). Then, ecl(K, ι, κ) ⊆ cl(G, ι, κ) and this implies cl(ecl(K, ι, κ), ι, κ) ⊆ cl(K, ι, κ). which contradicts to the relation (2). Hence the result.

Theorem 11 Let be SoIFTS. For ∈ and (ι,κ)∈*I*⊗*I*. It satisfies the following statements:

(i) K is (ι, κ)-fδpo ⇔ K = δpint(K, ι, κ),

(ii) K is (ι, κ)-fδpc ⇔ K = δpcl(K, ι, κ),

(iii) δpcl(0~, ι, κ) = ~0,

(iv) int(K, ι, κ) ⊆ δpint(K, ι, κ) ⊆ K ⊆ δpcl(K, ι, κ) ⊆ cl(K, ι, κ),

(v) δpcl(K, ι, κ) ∪ δpcl(L, ι, κ) ⊆ δpcl(K ∪ L, ι, κ),

(vi) δpcl(δpcl(K, ι, κ), ι, κ) = δpcl(cl(K, ι, κ), ι, κ) = cl(K, ι, κ).

Proof. It follows from Theorem 10.

Theorem 12 Let be SoIFTS. For ∈ and (ι,κ)∈*I*⊗*I*. It satisfies the following statements:

(i) K is (ι, κ)-fδso ⇔ K = δsint(K, ι, κ),

(ii) K is (ι, κ)-fδsc ⇔ K = δscl(K, ι, κ),

(iii) δscl(0~, ι, κ) = ~0,

(iv) int(K, ι, κ) ⊆ δsint(K, ι, κ) ⊆ K ⊆ δscl(K, ι, κ) ⊆ cl(K, ι, κ),

(v) δscl(K, ι, κ) ∪ δscl(L, ι, κ) ⊆ δscl(K ∪ L, ι, κ),

(vi) δscl(δscl(K, ι, κ), ι, κ) = δscl(cl(K, ι, κ), ι, κ) = cl(K, ι, κ).

Proof. It follows from Theorem 10.

Theorem 13 For an IFS *K* of a SoIFTSand (ι,κ)∈*I*⊗*I*, we have:

(i) eint(K, ι, κ)c = ecl(Kc, ι, κ),

(ii) ecl(K, ι, κ)c = eint(Kc, ι, κ),

Proof. (i) Since eint(K, ι, κ) ⊆ K and eint(K, ι, κ) is (ι, κ)-feo in X, Kc ⊆ eint(K, ι, κ)c and eint(K, ι, κ)c is (ι, κ)-fecin X. Thus ecl(Kc, ι, κ) ⊆ ecl(eint(K, ι, κ)c, ι, κ)= eint(K, ι, κ)c.

Conversely, since Kc ⊆ ecl(Kc, ι, κ) and ecl(Kc, ι, κ) is (ι, κ)-fec in X, ecl(Kc, ι, κ)c ⊆ K and ecl(Kc, ι, κ)c is (ι, κ)-feo in X. Thus ecl(Kc, ι, κ)c = eint(ecl(Kc, ι, κ)c, ι, κ)⊆ eint(K, ι, κ).

and hence eint(K, ι, κ)c ⊆ ecl(Kc, ι, κ).

(ii) Similar to (i).

Theorem 14 For an IFS *K* of a SoIFTS and (ι,κ)∈*I*⊗*I*, we have:

(i) δpint(K, ι, κ)c = δpcl(Kc, ι, κ),

(ii) δpcl(K, ι, κ)c = δpint(Kc, ι, κ),

Proof. It follow from Theorem 13.

Theorem 15 For an IFS *K* of a SoIFTS and (ι,κ)∈*I*⊗*I*, we have:

(i) δsint(K, ι, κ)c = δscl(Kc, ι, κ),

(ii) δscl(K, ι, κ)c = δsint(Kc, ι, κ),

Proof. It follow from Theorem 13.

IV. Conclusion

In the present paper we introduced fuzzy (ι,κ)-*e* (resp. (ι,κ)-δ pre and (ι,κ)-δ semi) open sets, their respective interior and closure operators on intuitionistic fuzzy topological spaces in Šostak's sense. We investigate some of their characteristic properties and establish the relations between them with some counter examples.

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