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Impact of Magnetic Field on the Creeping Flow Past a Slightly Deformed Sphere

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Abstract: An analytical solution is presented for the steady axisymmetric slow viscous flow of an electrically conducting fluid over a slightly deformed sphere under the influence of a transverse magnetic field using the Stokes approximation. The governing equations are changed into dimensionless form and resulting partial differential equations are solved using the method of separation of a variable. The stream function for the flow around the approximate sphere is calculated. As an application, we considered the electrically conducting fluid flow past an oblate spheroid and evaluated the drag force in the presence of a magnetic field. Some special cases are deduced from the present study and compared with some previous research. Plots of the non-dimensional drag with different fluid parameters are presented graphically. Also, the impact of the magnetic field on the patterns of streamlines has been discussed for different Hartmann number and deformation parameter. The outcome clarifies that the magnetic field enhances the drag on the oblate spheroid.

Keywords: Drag force, Hartmann number, Magnetic field, Stokes flow, Stream function.

I. Introduction

Over the past few years, many researchers have been focused to determine the significant and beneficial applications of magneto hydrodynamics. There are frequent daily life applications belong to the fields of magneto hydrodynamics in various areas such as biological science, astrophysics, geophysics, metallurgy, planetary atmospheres, etc.

Happel and Brenner [1] presented the problem of creeping flow over a rigid approximate sphere. Subsequently, this problem was investigated by Palaniappan [2] and Ramkissoon [3] with slip-stick boundary conditions. The creeping flow past a bit deformed fluid sphere has been analyzed by Ramkissoon [4]. Ramkissoon and Rahaman [5] have investigated the creeping motion of a wall effects on a rigid spherical particle using no-slip condition and drag experienced has been evaluated. The creeping motion over a porous approximate sphere has been analyzed by Srinivasacharya [6] and the similar work was examined by Srinivasacharya and Prasad [7] using stress jump boundary condition. Srinivasacharya [8] used slip boundary condition of Beavers and Joseph and Srinivasacharya and Prasad [9] applied stress jump boundary condition to solve the problem of the creeping flow past a porous approximate spherical shell. Gupta and Deo [10] have used Kuwabara boundary condition on the cell surface and evaluated the drag on the swarm of porous oblate spheroid. Jaiswal and Gupta [11] have investigated the Stokes flow of a Reiner-Rivlin fluid spheroid in a spherical container. Iyengar and Radhika [12] have investigated the slow viscous flow over a porous spheroid and calculated the drag on an oblate spheroid.

So many early studies (works) contributing to the field of magnetohydrodynamic have clearly explained its significance in science and engineering. A basic introduction of MHD flow can be found in the book [13]. Devi and Raghavachar [14] studied the magnetohydrodynamic stratified flow past a sphere. Stream functions for the creeping flow of conducting fluid past a sphere covered by a permeable spherical shell were calculated by Jayalakshmamma *et al.* [15]. Saad [16] applied unit cell models to analyze the quasisteady flow of an incompressible conducting fluid past an assemblage of porous spheres subject to a uniform transverse magnetic field. Prasad *et al.* [17] studied the impact of MHD flow past a semipermeable sphere particle and evaluated the drag experienced on the semipermeable sphere. Recently, Yadav [18] has investigated the effect of magnetic field on the creeping motion over porous spheroid: hydrodynamic permeability of a membrane using cell model technique. All foregoing works emphasize the effect of magneto hydrodynamics to be changing with varying shapes of the particle. Hence, it is important to study the MHD impact on varying particle geometry.

To the best of our knowledge, the MHD effect over a rigid or porous body of various shapes are available in the literature, but very few studies of MHD impact over a deformed spherical shape are available in the literature. Inspired by all of the foregoing analysis and gaps in the literature, we are interested in looking at the problem considering the MHD effect on the flow past an approximate spherical particle using an analytical approach. As a particular case, we considered the oblate spheroid and obtained the drag on the surface of the spheroid and its graphical results are obtained and discussed for different fluid parameters. Also, we derive some special cases and compared with some previous work whenever possible.

II. Problem Formulation and Solution

Fig. 1 illustrates axisymmetric, slow viscous flow of an electrically conducting fluid with uniform velocity U over a bit deformed sphere in the presence of the transverse magnetic field. Let the equation of approximate sphere which departs but a little in shape from a sphere $r = a$ be

$$r = a[1 + \sum \beta_m G_m(\zeta)] \tag{1}$$

where $G_m(\zeta)$ is the Gegenbauer function of the 1st kind of order m and degree -1/2. We assume the coefficient β_m is very small hence $O(\in^2)$ and its higher powers may be neglected i.e.

$$\left(\frac{r}{a}\right)^k \approx 1 + k\beta_m G_m(\zeta), \tag{2}$$

where, k may be $+ve$ or $-ve$ integer.

Magnetic force F is defined as $F = J \times H$ or $F = \mu_h^2 \sigma (q \times H) \times H$. Where J, H, μ_h and σ are the electric current density, magnetic field intensity, magnetic permeability and electric conductivity of the fluid respectively. The field equations governing the creeping flow of a viscous incompressible electrically conducting fluid are

$$\nabla . q = 0, \tag{3}$$

$$\nabla p + \mu \nabla \times \nabla \times q - \mu_h^2 \sigma (q \times H) \times H = 0, \tag{4}$$

Where p denotes pressure and q denotes velocity of the fluid and μ is the viscosity. The following dimensionless variables are used to make the governing equation in non-dimensional form

$$r = a\tilde{r},\ \ q = U\tilde{q},\ \ \nabla = \frac{\tilde{\nabla}}{a},\ \ p = \frac{\mu U}{a}\tilde{p},\ \ H = H_0 \tilde{H}. \tag{5}$$

Substituting these variables in (3) and (4) and omitting bar above the dimensionless variables, we get the equations

$$\nabla . q = 0, \tag{6}$$

$$\nabla p + \nabla \times \nabla \times q - \alpha^2 (q \times H) \times H = 0, \tag{7}$$

where $\alpha = \sqrt{\frac{\mu_h^2 H_0^2 \sigma a^2}{\mu}}$ is the Hartmann number. The azimuthal component of velocity $q_\phi = 0$ for axisymmetric creeping flow and the velocity components can be expressed as $q = q_r(r,\theta) e_r + q_\theta(r,\theta) e_\theta$. In axisymmetrical flow, the velocity components are related with stream function ψ as

$$q_r = -\frac{1}{r^2 sin\theta}\frac{\partial \psi}{\partial \theta},\ \ q_\theta = \frac{1}{r sin\theta}\frac{\partial \psi}{\partial r}. \tag{8}$$

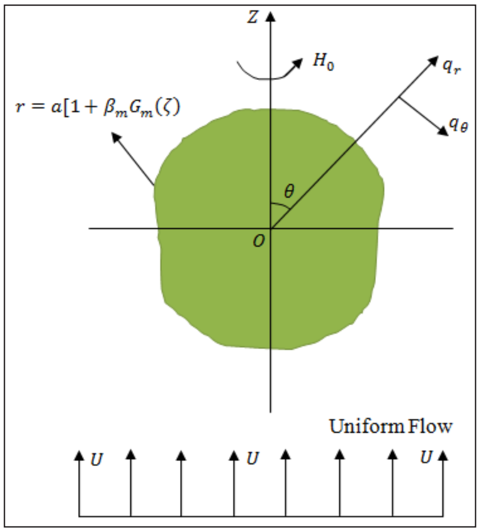

Fig. 1: Visualization of MHD Flow Past an Approximate Sphere

The pressure is eliminated in (7) and substituted velocity components from (8), we get

$$E^2(E^2 - \alpha^2)\psi = 0, \tag{9}$$

where, $E^2 = \frac{\partial^2}{\partial r^2} + \frac{(1-\zeta^2)}{r^2}\frac{\partial^2}{\partial \zeta^2},\ \ \zeta = \cos\theta.$

The non-singular of (9) in the flow region is obtained as

$$\psi = r^2 G_2(\zeta) + \sum_{n=2}^{\infty} \left[A_n r^{-n+1} + B_n \sqrt{r} K_{n-\frac{1}{2}}(\alpha r) \right] G_n(\zeta), \tag{10}$$

We can write (10) in the form

$$\psi = \left[r^2 + a_2 r^{-1} + b_2 \sqrt{r} K_{\frac{3}{2}}(\alpha r) \right] G_2(\zeta) + \sum_{n=3}^{\infty} \left[A_n r^{-n+1} + B_n \sqrt{r} K_{n-\frac{1}{2}}(\alpha r) \right] G_n(\zeta), \tag{11}$$

here a_2, b_2, A_n and B_n are the unknown constants to be determined by using different boundary conditions.

III. Boundary Conditions and Arbitrary Constants

The following boundary conditions are used on the surface of slightly deformed sphere,

- The impenetrability of the surface implies

$$q_r = 0 \quad i.e.\ \psi(r,\theta) = 0 \tag{12}$$

- The no slip at the surface implies

$$q_\theta = 0 \quad i.e.\ \psi_r(r,\theta) = 0 \tag{13}$$

Applying these above boundary conditions in (11), we obtain

$$\left[1 + a_2 + b_2 K_{\frac{3}{2}}(\alpha)\right] G_2(\zeta) + \left[2 - a_2 - b_2\left\{\alpha K_{\frac{1}{2}}(\alpha) + K_{\frac{3}{2}}(\alpha)\right\}\right] G_2(\zeta) G_m(\zeta)\beta_m + \sum_{n=3}^{\infty}\left[A_n + B_n K_{n-\frac{1}{2}}(\alpha)\right] G_n(\zeta) = 0, \tag{14}$$

$$\left[2 - a_2 - b_2\left(\alpha K_{\frac{1}{2}}(\alpha) + K_{\frac{3}{2}}(\alpha)\right)\right] G_2(\zeta) + \left[2 + 2a_2 + b_2(\alpha^2 + 2)K_{\frac{3}{2}}(\alpha)\right] G_2(\zeta) G_m(\zeta)\beta_m + \sum_{n=3}^{\infty}\left[(1 - n)A_n + B_n(nK_{n-\frac{1}{2}}(\alpha) - \alpha K_{n+\frac{1}{2}}(\alpha))\right] G_n(\zeta) = 0. \tag{15}$$

Equating the leading terms of (14) and (15) to zero, we get

$$1 + a_2 + b_2 K_{\frac{3}{2}}(\alpha) = 0, \tag{16}$$

$$2 - a_2 - b_2\left(\alpha K_{\frac{1}{2}}(\alpha) + K_{\frac{3}{2}}(\alpha)\right) = 0 \tag{17}$$

Solving (16) and (17) we get the value a_2 and b_2 as

$$a_2 = -1 - \frac{3K_{\frac{3}{2}}(\alpha)}{\alpha K_{\frac{1}{2}}(\alpha)} \ \ and \ \ b_2 = \frac{3}{\alpha K_{\frac{1}{2}}(\alpha)} \tag{18}$$

Substituting the values of a_2 and b_2 from (18) to (14) and (15) we get,

$$\sum_{n=3}^{\infty}\left[A_n + B_n K_{n-\frac{1}{2}}(\alpha)\right] G_n(\zeta) = 0 \tag{19}$$

$$\left[-\frac{3\alpha K_{\frac{3}{2}}(\alpha)}{K_{\frac{1}{2}}(\alpha)}\right] G_2(\zeta) G_m(\zeta)\beta_m + \sum_{n=3}^{\infty}\left[(1 - n)A_n + B_n(nK_{n-\frac{1}{2}}(\alpha) + \alpha K_{n+\frac{1}{2}}(\alpha))\right] G_n(\zeta) = 0 \tag{20}$$

To solve above two equations for remaining arbitrary constants A_n and B_n, we require the identity

$$G_m(\zeta) G_2(\zeta) = \frac{-(m-2)(m-3)}{2(2m-1)(2m-3)} G_{m-2}(\zeta) + \frac{m(m-1)}{(2m+1)(2m-3)} G_m(\zeta) - \frac{(m+1)(m+2)}{2(2m-1)(2m+1)} G_{m+2}(\zeta) \tag{21}$$

valid for $m > 2$. Using this identity in (19) and (20), for $n \neq m - 2, m, m + 2$, we get

$$A_n = B_n = 0 \tag{22}$$

and another system of equations in A_n and B_n, when n is equal to $m - 2$, m and $m + 2$, we get

$$\begin{bmatrix} 1 & K_{n-\frac{1}{2}}(\alpha) \\ 1 - n & nK_{n-\frac{1}{2}}(\alpha) - \alpha K_{n+\frac{1}{2}}(\alpha) \end{bmatrix} \begin{bmatrix} A_n \\ B_n \end{bmatrix} = \bar{a}_n \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \tag{23}$$

Where,

$$\bar{a}_n = \frac{n(n-1)\beta_m}{(2n+1)(2n-3)}, \ \Delta_1 = 0, \ \Delta_2 = \frac{3\alpha K_{\frac{3}{2}}(\alpha)}{K_{\frac{1}{2}}(\alpha)}$$

Solving (23), we get the values of A_n and B_n, which is

$$A_n = \frac{-3\alpha\beta_m b_n K_{\frac{3}{2}}(\alpha) K_{n-\frac{1}{2}}(\alpha)}{K_{\frac{1}{2}}(\alpha)\left[(2n-1)K_{n-\frac{1}{2}}(\alpha) - \alpha K_{n+\frac{1}{2}}(\alpha)\right]},$$

$$B_n = \frac{3\alpha\beta_m b_n K_{\frac{3}{2}}(\alpha)}{K_{\frac{1}{2}}(\alpha)\left[(2n-1)K_{n-\frac{1}{2}}(\alpha) - \alpha K_{n+\frac{1}{2}}(\alpha)\right]}$$

IV. Application to Oblate Spheroid

As an application of the foregoing analysis, we now take an oblate spheroid whose equatorial radius is d as

$$\frac{x^2+y^2}{d^2} + \frac{z^2}{d^2(1-\epsilon)^2} = 1 \tag{24}$$

where $\in$ is very small quantity, therefore we are neglecting its squares and higher powers. The polar equation of (24) is

$$r = a(1 + 2\epsilon\, G_2(\zeta)) \tag{25}$$

where $a = d(1 - \in)$. Comparing Eq. (25) of oblate spheroid with (1) of approximate sphere, we find that $\beta_2 = 2\in$ and $\beta_m = 0$ when $m \neq 2$. Substituting these values in (11), we get the stream function as

$$\psi = \left(r^2 + (a_2 + A_2)r^{-1} + (b_2 + B_2)\sqrt{r}\, K_{\frac{3}{2}}(\alpha r)\right) G_2(\zeta) + \left(A_4 r^{-3} + B_4\sqrt{r}\, K_{\frac{7}{2}}(\alpha r)\right) G_4(\zeta) \tag{26}$$

where A_2, B_2, A_4, B_4 are unknown constants whose expressions are given by

$$A_2 = \left[\frac{6\alpha K_{\frac{3}{2}}(\alpha) K_{\frac{3}{2}}(\alpha)\beta_2}{5K_{\frac{1}{2}}(\alpha)\left(3K_{\frac{3}{2}}(\alpha) - \alpha K_{\frac{5}{2}}(\alpha)\right)}\right],$$

$$B_2 = \left[\frac{-6\alpha K_{\frac{3}{2}}(\alpha)\beta_2}{5K_{\frac{1}{2}}(\alpha)\left(3K_{\frac{3}{2}}(\alpha) - \alpha K_{\frac{5}{2}}(\alpha)\right)}\right],$$

$$A_4 = \left[\frac{4\alpha K_{\frac{3}{2}}(\alpha) K_{\frac{7}{2}}(\alpha)\beta_4}{5K_{\frac{1}{2}}(\alpha)\left(7K_{\frac{7}{2}}(\alpha) - \alpha K_{\frac{9}{2}}(\alpha)\right)}\right],$$

$$B_4 = \left[\frac{-4\alpha K_{\frac{3}{2}}(\alpha)\beta_4}{5K_{\frac{1}{2}}(\alpha)\left(7K_{\frac{7}{2}}(\alpha) - \alpha K_{\frac{9}{2}}(\alpha)\right)}\right] \quad (27)$$

V. Determination of Drag on Oblate Spheroid

A formula of drag force on sphere for creeping flow of magneto viscous fluid is suggested by [16, 17]

$$F_D = \pi\mu U a \int_0^{\pi} \left[r^4 \sin^3\theta \frac{\partial}{\partial r}\left(\frac{E^2\psi}{r^2\sin^2\theta}\right) - \alpha^2 r^2 sin\theta \frac{\partial\psi}{\partial r}\right]_{r=(1+\beta_2 G_2(\zeta))} d\theta \quad (28)$$

Substituting the value ψ of from (26) to (28) and carrying out the integration with respect to θ, we get

$$F_D = \frac{2}{3}\pi\mu U a \alpha^2 \left[-2 + a_2 + A_2 - 2(b_2 + B_2)K_{\frac{3}{2}}(\alpha) + \frac{4}{5}\alpha\,\beta_2\, b_2\, K_{\frac{1}{2}}(\alpha) - \frac{12}{5}\beta_2\right] \quad (29)$$

Substituting the values a_2, b_2, A_2 and B_2 and $a = d(1 - \in)$, $\alpha = \alpha_1(1 - \in)$ and in (29), we obtain the drag force in the presence of MHD impact.

$$F_D = (-2\pi\mu U d)\left[3 + 3\alpha_1 + 3\alpha_1^2 - \frac{1}{5}\epsilon(3 + 6\alpha_1 + 3\alpha_1^2)\right] \quad (30)$$

where $\alpha_1 = \sqrt{\frac{\mu_h^2 H_0^2 \sigma d^2}{\mu}}$

Special Cases

- If $\in = 0$ in (30), the approximate sphere geometrically becomes a perfect sphere with magnetic field and the drag is

$$F_D = -2\pi\mu U d(\alpha_1^2 + 3\alpha_1 + 3) \quad (31)$$

The above result agrees with the work of Prasad and Bucha [17].

- If magnetic fields $H_O \to 0$ i.e. $\alpha \to 0$ then the drag reduce is

$$F_D = -6\pi\mu U d\left[1 - \frac{1}{5}\epsilon\right] \quad (32)$$

This is the result earlier reported by Happel and Brenner [1].

- If $\in = 0$ in (32), the renowned Stokes formula past a perfect sphere [1] is obtained

$$F_D = -6\pi\mu U d \quad (33)$$

VI. Results and Discussion

Non-dimensional drag D_N is defined as the ratio of the drag force F_D to the Stokes force $F_{st} = -6\pi\mu U d$ as follows:

$$D_N = \frac{F_D}{-6\pi\mu U d} \quad (34)$$

The drag coefficient D_N is plotted for numerous values of Hartmann number(α), deformation parameter(ε) and its variation are presented in Fig. 2 and 3. Fig. 2 shows the plot of the non-dimensional drag D_N with α for numerous values of ε. From Fig. 2 it is noticed that the non-dimensional drag D_N increases when Hartmann number α increases. Also, we observed that the oblate spheroid experiences more amount of resistance of flow in comparison to perfect sphere.

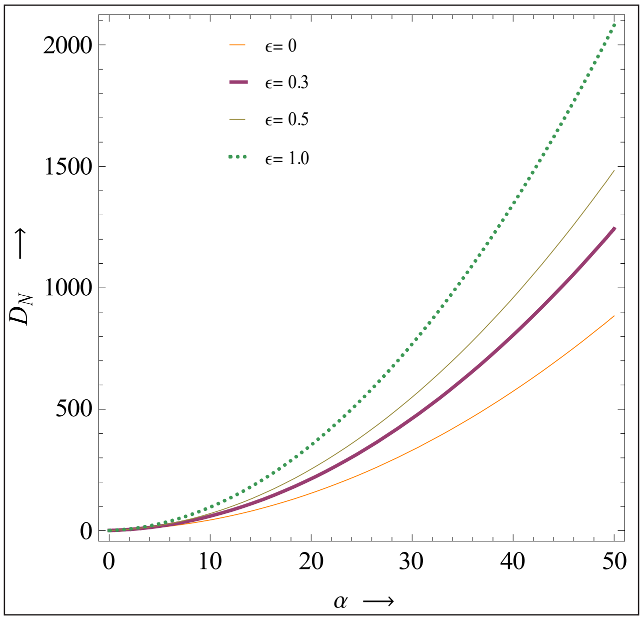

Fig. 2: Representing D_N versus Hartmann Number α for Numerous Values of Deformation Parameter $\in$

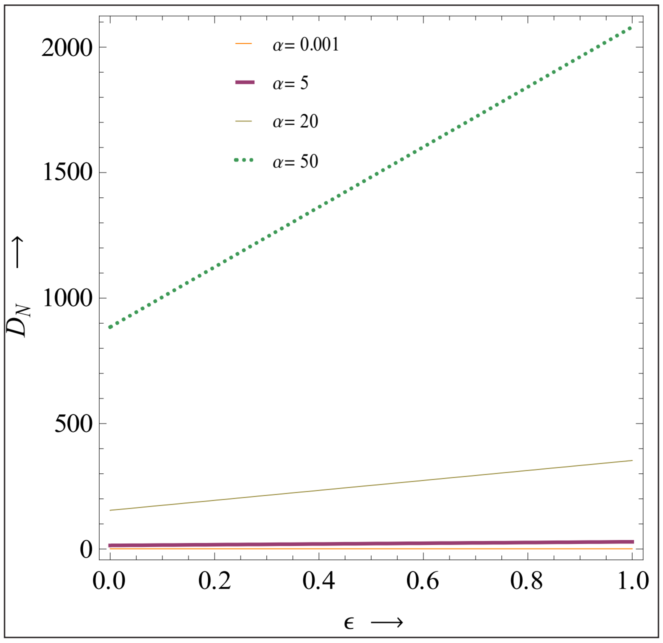

Fig. 3: Representing D_N versus Deformation Parameter $\in$ for Numerous Values of Hartmann Number α

The influence of deformation parameter on drag coefficient D_N for different values of is shown in Fig. 3. It is seen in Fig. 3 that the drag coefficient D_N increases as the deformation parameter $\in$ increases. The Hartmann number enhances drag on the surface of the deformed sphere for every value of deformation parameter. Also, we observed that the small value of Hartmann number the drag is constant for every value of deformation parameter.

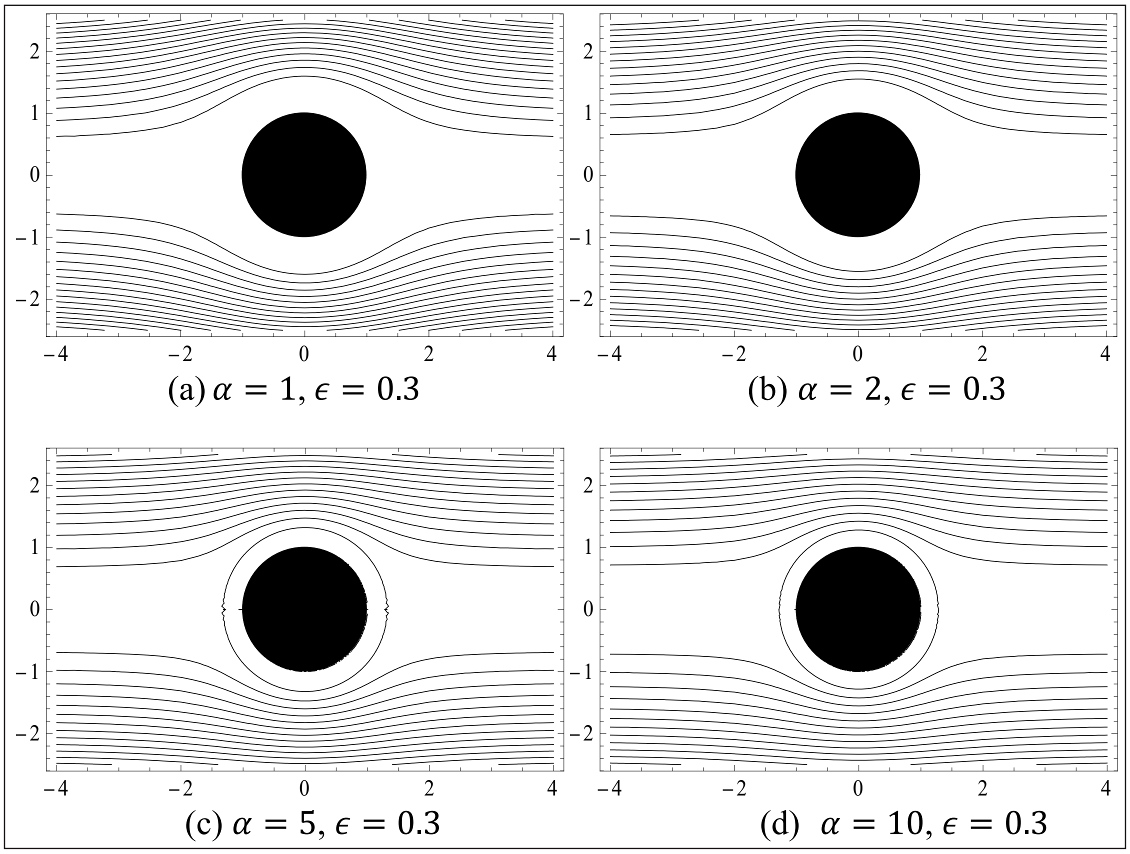

Fig. 4 (a)-(d): Representing Streamlines versus Hartmann Number

The effect of Hartmann number α for fixed deformation parameter $\in$ on the streamlines is shown in Fig. 4(a)-(d). For $\in = 0.3$ and for small Hartmann number $\alpha = 1$ and 2, it is observed that the streamlines are pass passing away from the oblate spheroid, shown in Fig. 4(a) and (b). For the same, when the Hartmann number strength is increased, the fluid starts to move over the oblate spheroid. As a result the streamlines are moving closer to the surface of the oblate spheroid and the same is illustrated in Fig. 4(c) and (d). From the graph, the meandering of streamlines over the surface of the oblate spheroid is observed for the increase in Hartmann number strength with fixed deformation parameter.

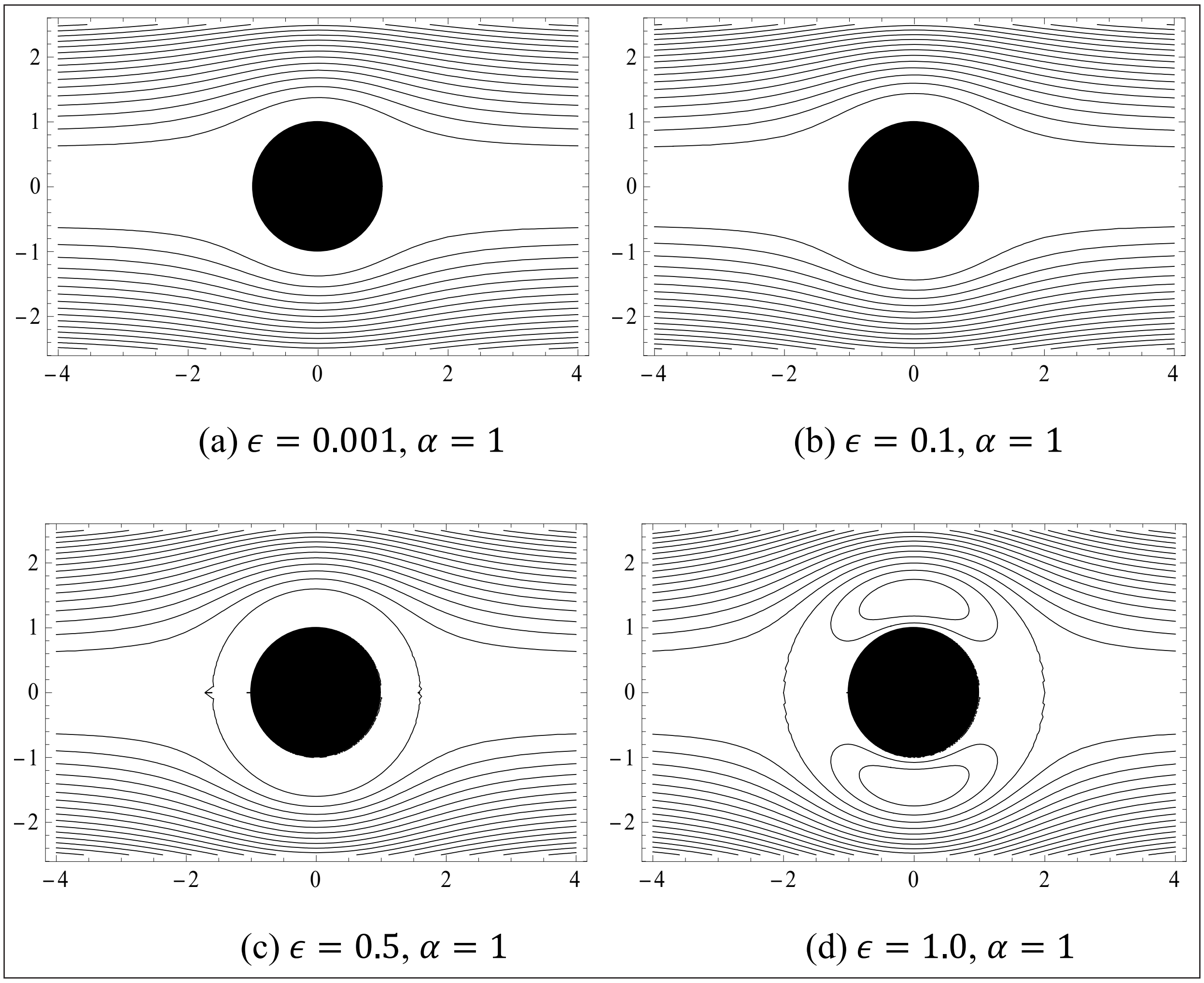

Fig. 5 (a)-(d): Representing Streamlines versus Deformation Parameter

Fig. 5(a)-(d) shows the impact of fluid flow for fixed value of Hartmann number $\alpha = 1$ and varying value of deformation parameters $\in = 0.001, 0.1, 0.5, 1.0$. For $\alpha = 1$ and $\in = 0.001$ and 0.1, it is noticed that the fluid is flowing away from the oblate spheroid shown in Fig. 5(a)-(b). As a deformation parameter increases for the fluid is flowing over the oblate sphere and streamlines are closer to the surface of the oblate spheroid, as illustrated in Fig. 5(c)-(d).

VII. Conclusion

The stream function for Stokes flow of viscous fluid over a slightly deformed sphere under the influence of a transverse uniform magnetic field has been obtained. We have calculated the drag force for the oblate spheroid and its variations has been plotted for different parameters of fluid. It is observed that the magnetic field enhances drag on the surface of the deformed sphere for every value of the deformation parameter. The oblate sphere experiences more amount of resistance of flow in comparison to a perfect sphere. The effects of Hartman number and deformation parameter on streamlines are analyzed and it is observed that streamlines are attracted towards the oblate spheroid when Hartman number and deformation parameter increases.

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