

Reliability Determination of a Complex Engineering Item Under the Distributional Setup

Sadananda Chatterjee

Rabindra Mahavidyalaya, Champadanga, Hooghly, West Bengal, India.

Email: sadanandastat@gmail.com

Abstract: This paper proposes a technique for finding reliability of a complex system in terms of design parameters where exact analytical determination of reliability is difficult to manage. Here we have established reliability approximation of tensile stress of an element under the Weibull setup. Numerical studies of this approach have also been cited. Numerical studies indicate that the proposed technique gives very good approximation of reliability of complex systems under stress-strength set-up. This technique is conceptually simple, handles analytic intractability and reduces computational time. This technique can be employed in manufacturing industries for production of high-reliable items.

Keywords: Stress strength model, Reliability bounds, Approximation, Extent of error, Engineering Item.

I INTRODUCTION

To find the system reliability, the application of stress strength model, is gaining momentum in the hands of reliability engineers during the early stages of product design. Under this model reliability is defined as the probability of the event that random stress (S) is less than the random strength (Y). If we know the stress and strength distribution then reliability can be computed in term of the parameters of those distributions. If we know the stress and strength distributions, ordinary transformation techniques due to Parzen [1] can be applied for the computation of system reliability. When stress and strength variables are made of multiple stochastic factors then problem arises in computation of reliability. The inference theory approach for reliability prediction due to kapur and Lamberson [2] fails because for a complex system stress or strength distributions are mostly unknown.

Taylor-series method, Monte-Carlo method, Quadrature method, Discretization Method and Discrete concentration method are existing approaches for estimating reliability of complex engineering items. To approximate the stress or strength distribution, Taguchi [3] proposed a factorial experiment approach which discretizes a continuous variable by a 3-point discrete distribution. D'Errico and Zaino [4] modified Taguchi's concept for discretizing stress and strength variables based on moment equalization. English et al [5] applied this moment equalization rule in the stress-strength setup. Roy and Ghosh [6] proposed to discretize a continuous random variable by equating the raw

moments of the original distribution and they discretized distribution. Discretization of Exponential distribution was carried out by them. Barbiero [7] proposed a discretizing method for reliability estimation in complex stress-strength model. Roy and Dasgupta [8] depicted discretizing procedure using discrete concentration of Roy [9] and they gave a new approximating technique by using survival function. This method is mentioned as method of discrete concentration. Roy and Dasgupta [10] suggested a discretization approach by using a survival function of a continuous random variable. approach for the Weibull distribution also. Ghosh et al [12] depicted the usefulness of discretization of a random variable using the reversed hazard rate function.

Nayak and Roy [13] described bound based reliability approximation under the Weibull, Rayleigh and Exponential setups for simplified form of I-beam.. Nayak and Roy [14] also studied a new approach for approximating reliability of a complex system under the Weibull setup. Here they approximated reliability of I-beam. Nayak [15] described reliability approximation of hollow rectangular tube under the Weibull and Rayleigh setups. Nayak et al [16] computed reliability of solid shaft under the Gamma set up. Nayak and Roy [17] also depicted bound based reliability approximation of an engineering item, resistor under the stress-strength model. Nayak and Seal [18] has determined reliability values of Ball bearing under the Weibull frame work.

Here we have proposed reliability approximation of tensile stress under the Weibull set up. Extent of error is also established for this important engineering item.

II RELIABILITY APPROXIMATION APPROACH

Finding of exact reliability of a complex system is mostly analytic hard. We have already mentioned that some techniques are available in the literature for approximating the reliability for intractable cases. But some drawbacks are observed in their work. All the approaches, mentioned above suffer from locking effects in the sense that the approximated reliability values remain constant towards changes in the value of the strength parameter. The author's have concentrated on simulation study. But further manipulation in terms of design parameter can't be under taken under their approaches. There are cases where discrete

approximations are extremely weak. For instance, under the Exponential setup where lack of memory property holds, the discretization approach doesn't offer close approximate value.

So to bridge this gap of this field, we propose an efficient approach to approximate reliability of complex engineering items accounting for competitor pricing reaction. The current proposed work aims at offering a different approach for reliability approximation so that one not only gets a clear idea about the extent of error but also can manipulate reliability in terms of design parameters. Our approach can be applied in manufacturing industries for producing high reliability items. Actually this approach will help to improve the manufacturing industries. Our proposed procedure can be implemented to other methods available in the literature for the evaluation of reliability in stress strength model, where stress and strength are two functions of different stress and strength r.v. components. Our proposed approach is as follows Here we have proposed average of the two bounds as the reliability approximation and half of the absolute deviation between the two bounds as the extent of error. It is a crucial alternative in approximating system reliabilities of complex system where analytic methods fail to offer a closed-form solution. Our proposed reliability approximation is a function of the design parameters and the corresponding error-bound can be easily calculated. Let U and L are the upper and lower reliability bounds respectively. The corresponding reliability approximation, R_{approx} , and extent of error are given as follows:

$$R_{approx} = \frac{U+L}{2}$$

The extent of error is given by

$$\text{Error} = |R - R_{approx}| = |R - \frac{(U+L)}{2}|$$

Since $R \leq U$

$$R - \frac{(U+L)}{2} \leq U - \frac{(U+L)}{2} = \frac{(U-L)}{2}$$

Similarly, $R - \frac{(U+L)}{2} \geq L - \frac{(U+L)}{2} = \frac{(L-U)}{2}$

Therefore, $|R - \frac{(U+L)}{2}| \leq \frac{(U-L)}{2}$

Thus, $\text{Error} \leq \frac{(U-L)}{2}$

On the basis of distributional assumption, we will determine upper and lower bounds of reliability of tensile stress. So, Extent of error will be obtained in terms of distributional parameters.

III. RELIABILITY BOUNDS UNDER THE WEIBULL SETUP

Weibull set up would be more appropriate description of any engineering item as it provides an excellent way to focus on both burn-in and burnout phenomena and can model increasing failure rate (IFR) and decreasing failure rate (DFR) class of distribution for different choices of shape parameter. This distribution is the most widely used stochastic description of the life distribution of a component [19], for stress and strength variables. As a result, a study on bound based reliability approximation based on Weibull setup will have a wider appeal.

The tensile Stress of the element is given by

$$S = \frac{P}{\pi r^2}, [2]$$

Where, P is the load acting on the element and r is the radius of the circular cross section. Now we are interested to find reliability of tensile stress of an item. When Y is the Strength and S is the stress then under the stress-strength model, reliability, R, is given by

$$\begin{aligned} R &= P(Y > S) \\ &= P(Y > \frac{P}{\pi r^2}) \\ &= P(r > (\frac{P}{\pi Y})^{\frac{1}{2}} = E_P E_R [P\{(\frac{P}{\pi Y})^{\frac{1}{2}} | P = p \text{ and } R = r\}] \end{aligned}$$

$$\text{Therefore, } R = \int_0^\infty \int_0^\infty e^{-\gamma(\frac{P}{\pi Y})^{\frac{\beta}{2}}} dF_Y dF_P \tag{1}$$

Here we have assumed that stress component P follows $W(\rho, \psi)$ and stress component r follows $W(\gamma, \beta)$. We have also assumed that strength Y follows $W(\theta, \mu)$. We have also assumed that strength and stress components are independent. Under this distributional background, we are interested to find reliability bounds.

Note that $E(P^r) = \frac{\Gamma(1+\frac{r}{\psi})}{\rho^{\frac{r}{\psi}}}$ and $E(P^{-r}) = \frac{\Gamma(1-\frac{r}{\psi})}{\rho^{-\frac{r}{\psi}}}$ with the survival function $S_P(p) = e^{-\rho P^\psi}$

Result 1: Reliability upper bound, S ($\gamma, \beta, \psi, \rho, \mu, \theta$), when Tensile stress and strength of an element follow Weibull distribution is given by

$$S(\gamma, \beta, \psi, \rho, \mu, \theta) = 1 - \frac{\gamma}{\pi^2} \frac{\Gamma(1+\frac{\beta}{2\psi})}{\rho^{\frac{\beta}{2\psi}}} \frac{\Gamma(1-\frac{\beta}{2\mu})}{\theta^{\frac{-\beta}{2\mu}}} + \frac{\gamma^2}{2\pi\beta} \frac{\Gamma(1+\frac{\beta}{\psi})}{\rho^{\frac{\beta}{\psi}}} \frac{\Gamma(1-\frac{\beta}{\mu})}{\theta^{\frac{-\beta}{\mu}}}$$

for $\mu > \beta$.

Proof: Note that $e^{-l} \leq 1 - l + \frac{l^2}{2}$ for $l > 0$.

Now from (i) with the choice of $l = \gamma(\frac{P}{\pi Y})^{\frac{\beta}{2}}$, we get

$$\begin{aligned} R &\leq \int_0^\infty \int_0^\infty [1 - \gamma(\frac{P}{\pi Y})^{\frac{\beta}{2}} + \frac{\gamma^2}{2} (\frac{P}{\pi Y})^\beta] dF_Y dF_P \\ &= 1 - \frac{\gamma}{\pi^2} E(P^{\frac{\beta}{2}}) E(Y^{-\frac{\beta}{2}}) + \frac{\gamma^2}{2\pi\beta} E(P^\beta) E(Y^{-\beta}) \\ &= 1 - \frac{\gamma}{\pi^2} \frac{\Gamma(1+\frac{\beta}{2\psi})}{\rho^{\frac{\beta}{2\psi}}} \frac{\Gamma(1-\frac{\beta}{2\mu})}{\theta^{\frac{-\beta}{2\mu}}} + \frac{\gamma^2}{2\pi\beta} \frac{\Gamma(1+\frac{\beta}{\psi})}{\rho^{\frac{\beta}{\psi}}} \frac{\Gamma(1-\frac{\beta}{\mu})}{\theta^{\frac{-\beta}{\mu}}} \text{ for } \mu > \beta \tag{2} \end{aligned}$$

Therefore, Reliability upper bound, S ($\gamma, \beta, \psi, \rho, \mu, \theta$), when Tensile stress and strength of an element follow Weibull distribution is given by

$$S(\gamma, \beta, \psi, \rho, \mu, \theta) = 1 - \frac{\gamma}{\pi^2} \frac{\Gamma(1+\frac{\beta}{2\psi})}{\rho^{\frac{\beta}{2\psi}}} \frac{\Gamma(1-\frac{\beta}{2\mu})}{\theta^{\frac{-\beta}{2\mu}}} + \frac{\gamma^2}{2\pi\beta} \frac{\Gamma(1+\frac{\beta}{\psi})}{\rho^{\frac{\beta}{\psi}}} \frac{\Gamma(1-\frac{\beta}{\mu})}{\theta^{\frac{-\beta}{\mu}}}$$

, for $\mu > \beta$.

Result 2: Reliability lower bound, I ($\gamma, \beta, \psi, \rho, \mu, \theta$), when Tensile stress and strength of an element follow Weibull distribution is given by

$$I(\gamma, \beta, \psi, \rho, \mu, \theta) = 1 - \frac{\gamma}{\pi^2} \frac{\Gamma(1+\frac{\beta}{2\psi})}{\rho^{\frac{\beta}{2\psi}}} \frac{\Gamma(1-\frac{\beta}{2\mu})}{\theta^{\frac{-\beta}{2\mu}}}, \text{ for } \mu > \beta.$$

Proof: Note that $e^{-l} \geq 1 - l$ for $l > 0$.

Now from (i) with the choice of $l = \gamma(\frac{P}{\pi Y})^{\frac{\beta}{2}}$, we get

$$R \geq \int_0^\infty \int_0^\infty [1 - \gamma(\frac{P}{\pi Y})^{\frac{\beta}{2}}] dF_Y dF_P$$

$$= 1 - \frac{\gamma}{\pi^2} E(P^{\frac{\beta}{2}}) E(Y^{-\frac{\beta}{2}})$$

$$= 1 - \frac{\gamma}{\pi^2} \frac{\Gamma(1+\frac{\beta}{2\psi}) \Gamma(1-\frac{\beta}{2\mu})}{\rho^{2\psi} \theta^{2\mu}} \text{ for } \mu > \beta \tag{3}$$

Therefore, Reliability lower bound, $I(Y, \beta, \psi, \rho, \mu, \theta)$, when Tensile stress and strength of an element follow Weibull distribution is given by

$$I(Y, \beta, \psi, \rho, \mu, \theta) = 1 - \frac{\gamma}{\pi^2} \frac{\Gamma(1+\frac{\beta}{2\psi}) \Gamma(1-\frac{\beta}{2\mu})}{\rho^{2\psi} \theta^{2\mu}}$$

IV RELIABILITY ESTIMATION AND EXTENT OF ERROR UNDER THE WEIBULL SET UP:

When the determination of actual reliability becomes intractable we make use of approximation approach. Here we have considered mean of two bounds as the reliability approximation and half of the absolute difference between the two bounds as the extent of error. Therefore, on the basis of result1 and result2, reliability approximation of tensile stress is given by

$$R(Y, \beta, \psi, \rho, \mu, \theta) \leq \frac{S(Y, \beta, \psi, \rho, \mu, \theta) + I(Y, \beta, \psi, \rho, \mu, \theta)}{2} = 1 -$$

$$\frac{\gamma}{\pi^2} \frac{\Gamma(1+\frac{\beta}{2\psi}) \Gamma(1-\frac{\beta}{2\mu})}{\rho^{2\psi} \theta^{2\mu}} + \frac{\gamma^2}{4\pi^2} \frac{\Gamma(1+\frac{\beta}{2\psi}) \Gamma(1-\frac{\beta}{2\mu})}{\rho^{2\psi} \theta^{2\mu}} \text{ for } \mu > \beta. \tag{4}$$

This reliability approximation approach is simple to use and having intuitive appeal, is expected to be preferred by the design engineers. Again using result1 and result2, extent of error in terms of design parameter of tensile stress is given by

$$E(Y, \beta, \psi, \rho, \mu, \theta) \leq \frac{S(Y, \beta, \psi, \rho, \mu, \theta) + I(Y, \beta, \psi, \rho, \mu, \theta)}{2} = 1 -$$

$$\frac{\gamma}{\pi^2} \frac{\Gamma(1+\frac{\beta}{2\psi}) \Gamma(1-\frac{\beta}{2\mu})}{\rho^{2\psi} \theta^{2\mu}} \text{ for } \mu > \beta. \tag{5}$$

V STUDY OF CLOSENESS BETWEEN THE TWO RELIABILITY BOUNDS

We have approximated the reliabilities of the element at different values of γ using our proposed reliability approximation technique. For a numerical evaluation of reliability bounds, reliability values and extent of error of tensile stress, we choose the parameters in such a way so that the extent of error becomes less. For this purpose we have taken $\beta = 10, \psi = 5, \rho = 6, \mu = 18, \theta = 7$ and the parameter, γ , has been allowed to vary.

One may observe from the given table that the upper and lower reliability bounds are reasonably close, especially for high reliability values. Thus for the case of high reliability value, midpoint of these bounds can fairly approximate the actual value itself.

Table 1: Showing system reliability and extent of error for tensile stress under the proposed approach

Sl.No.	γ	Upper bound	Lower bound	Reliability approximation	Extent of error
1	780	0.809346	0.687832	0.748589	0.060757
2	760	0.811198	0.695836	0.753517	0.057681
3	740	0.813211	0.703841	0.758526	0.054685
4	720	0.815383	0.711845	0.763614	0.051769
5	710	0.816529	0.715847	0.766188	0.050341
6	700	0.817715	0.719849	0.768782	0.048933
7	690	0.818941	0.723851	0.771396	0.047545
8	680	0.820207	0.727854	0.774030	0.046177
9	670	0.821513	0.731856	0.776684	0.044829
10	660	0.822859	0.735858	0.779358	0.043500
11	520	0.845894	0.791888	0.818891	0.027003
12	500	0.849824	0.799892	0.824858	0.024966
13	475	0.854961	0.809898	0.832429	0.022532
14	450	0.860348	0.819903	0.840125	0.020222
15	425	0.865984	0.829908	0.847946	0.018038
16	400	0.871870	0.839914	0.855892	0.015978
17	375	0.878006	0.849919	0.863963	0.014043
18	350	0.884391	0.859925	0.872158	0.012233
19	325	0.891026	0.86993	0.880478	0.010548
20	300	0.897911	0.879935	0.888923	0.008988
21	275	0.905045	0.889941	0.897493	0.007552
22	250	0.912429	0.899946	0.906188	0.006241
23	225	0.920063	0.909952	0.915007	0.005056

24	200	0.927946	0.919957	0.923951	0.003995
25	175	0.936079	0.929962	0.933021	0.003058
26	150	0.944462	0.939968	0.942215	0.002247
27	125	0.953094	0.949973	0.951533	0.001560
28	100	0.961976	0.959978	0.960977	0.000999
29	80	0.969261	0.967983	0.968622	0.000639
30	70	0.972964	0.971985	0.972474	0.000489
31	60	0.976706	0.975987	0.976347	0.000360
32	50	0.980489	0.979989	0.980239	0.00025
33	40	0.984311	0.983991	0.984151	0.00016
34	30	0.988173	0.987994	0.988083	8.99E-05
35	20	0.992076	0.991996	0.992036	3.99E-05
36	15	0.994042	0.993997	0.994019	2.25E-05
37	10	0.996018	0.995998	0.996008	9.99E-06
38	5	0.998004	0.997999	0.998001	2.50E-06
39	4	0.998402	0.998399	0.998401	1.60E-06
40	3	0.998801	0.998799	0.998800	8.99E-07
41	2	0.999200	0.999200	0.999200	3.99E-07
42	1	0.999600	0.999600	0.999600	9.99E-08

VI IMPORTANCE OF THE PROPOSED TECHNIQUE

This reliability approximation technique can be employed in designing engineering items. This technique will reduce overuse of resource and replacement costs of the manufacturing industries for producing these items. Therefore, production of high reliable items can be made using this technique. Hence, we can say that use of this technique will result financial growth and good reputation of the manufacturing industries.

This technique is easy to understand and use and has greater applicability because reliability approximation and extent of error will be obtained in terms of distributional parameters so that one can increase or decrease reliability according to their requirements.

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