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PREDICTION AND OPTIMIZATION OF SPUR GEAR PAIR BY RESPONSE SURFACE METHOD

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ABSTRACT

Spur gear design is one of complex and time consuming design procedure. It will be great if it is automated by computer application. We can so reduce human error and make process faster. This paper presents development of mathematical models to predict module, number of teeth of pinion and gear of medium size external spur gear pair of 20 degree full depth teeth. This paper presents unique method to investigate engineering problem, its analysis, mathematical modelling and optimization with the help of RSM-response surface methodology and design of experiments (DOE). In the first part of our project we developed software which will assist in designing spur gear pair. We are taking dimensions as observations generated at the end. The second part constitutes study of relations between variables and development of mathematical models to predict dimensions directly without following design procedure. Finally we worked on optimization of design. Observations obtained from models are in compromising accuracy with actual design observations.

I. INTRODUCTION

Spur gear design takes lot of factors into consideration. Lot of research work has been done on profile behavior, analysis of composite profiles, wear reduction, reduction in transmission error, vibration analysis, compact gearbox designs and root-stress analysis etc. to understand different phenomenon of spur gears. Our objective is to study and observe different interaction effects of variables of spur gear design by RSM- response surface method and optimize it. Here we are finding center distance at which we can get minimum number of teeth, maximum module and at the same time maximum power transmission capability. Initially we designed a computer program which can design a pair of external spur gears of medium size to observe variations between different variables.

DOE-design of experiments method has been used to take observations. Observations are then analyzed and different graphs have been plotted for mathematical model. Single as well as global objective functions to optimize design variables were used.

2. LITERATURE REVIEW

G. Cockerham [1] developed a program for designing spur gear of 20 degree pressure angle. The program was fully integrated, requiring only design specifications and material properties to be supplied. B.S.Tong et.al.[2] also presented an interactive program to design internal spur gear pairs. From a specification, the program first performs a kinematic analysis to determine teeth numbers and to satisfy centre distance requirements.

B.S. Tong and D. Walton [3] worked on set of design variables are defined in terms of the number of pinion and annulus teeth and the module. The objective functions of minimum centre distance and volume were expressed. Some special search strategies were presented in order to solve the problems of a discrete number of variables and to reduce the calculation time. Hunglin Wanga et.al. [4] described in his paper, the mathematical formulation

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and an algorithmic procedure to solve this multiple objective gear design problem.

To design and optimize multi-spindle gear trains of the non speed change type gear, S. Prayoonrat and D. Walton [5] proposed another algorithm. The designer may choose to optimize gear trains on the basis of minimum overall centre distance, minimum overall size, minimum gear volume, or other desirable criteria, such as maximum contact or overlap ratios. The method is based on a two-stage optimization procedure. A direct search method was used.

Takao Yokota [6] optimized weight of gear by genetic algorithm. Similarly Cevdet Gologlua et.al.[7] automated preliminary design by genetic algorithm. Relation between wear and tooth width modification is spur gear was well presented by Huseyin et.al [8].

3. PLAN OF INVESTIGATION

3.1 Identification of Process Parameter

3.2 Development of Spur Gear Design Program on C++.

3.3 Finding the limits of the Process Parameters.

3.4 Development of Design Matrix and Conduction Experiments.

3.5 Development of Mathematical Models.

3.6 Checking Adequacy of Models (ANOVA).

3.7 Development of Final Model.

3.1 Identification of Process Parameter

Center distances, rpm of pinion and gear and power source are independent factors. While module, number of teeth are depending upon above four factors and depends on each other.

According to M.F. Spotts [10] beam strength is given by S_b= m*b*ó*y, where m = module, b = tooth width, ó = bearing stress (= ultimate tensile strength/ 3). Our objective is to maximized module to increase beam strength because of direct proportionality and to minimize number of teeth which is inversely proportional to module.

While other geometrical factors like addendum, dedendum, tooth thickness are according to empirical standard relationship. But they are not considered as deciding factors for gear design. So module, no. of teeth on pinion and no. of teeth on ear are response factors and shaft distance, rpm of pinion, rpm of gear, power output are input factors.

3.2 Development of Spur Gear Design Program

A spur gear design program is developed in C++ coding language. Spur gear beam strength, wear strength, surface hardness etc. is calculated by the program and finally it gives dimensions of gear pair. Inputs given to the program are center distance, rpm of pinion (smaller) and gear (bigger) and power. Other inputs are material elasticity, factor of safety, pressure angle etc. Program consists of inbuilt design data tables. Required values are chosen by program itself wherever necessary during execution. Inbuilt design data tables are user material selection, form factor selection, grade of gears, deformation constant and pitch error calculation [9]. Program is developed to design spur gears according to ISO 6336 (B) and AGMA standards [10]. Program shows failure of design when dynamic loads are equal or more than the beam strength or wear strength. If in any case during program execution design fails, then user can change module and manufacturing grade number to design foolproof gear pair.

Range of input design parameters is follows: power variation from 5 to 9 kW, shaft center distance varies between 200 to 300 mm, rpm of pinion varies between 1000 to 1400 rpm, and for gear it varies within 250 to 350 rpm. Other constant design parameters are starting torque multiplication factor = 1.2, bearing stress = 200 N/mm^2. Factor of safety = 1.5, Grade of gears = 6, Tooth form = 20^0 full depth, Material for both pinion and gear is steel, Elastic limit 206000 N/mm^2, tooth width = 10 *m.

3.3 Finding the Limits of the Process Parameters

Trial runs were carried out by varying one of the process parameters while keeping the rest of them at constant values. The upper limit of a factor was coded as +2 and the lower limit as -2. The coded values for intermediate values were calculated from the following relationship given below: Xi = 2 [2X - (Xmax + Xmin)] / [Xmax – Xmin] Where Xi is the required coded value of the variable X; X is any value of the variable from Xmin to Xmax; Xmin is the lower level of the variable & Xmax is the upper level of the variable. The process parameters levels with their units and notations are given in table1.

Factor	Unit	Sym	Factor levels				
			-2	-1	0	1	2
Shaft dist.	mm	SD	200	225	250	275	300
Rpm of pinion	rev/min	RP	1000	1100	1200	1300	1400
Rpm of gear	rev/min	RG	250	275	300	325	350
Power	kW	PO	5	6	7	8	9

TABLE 1

3.4 Developing Design Matrix and Conducting Experiments

The selected design matrix, shown in table 2, is four factors, five levels composite rotatable factorial design was selected for conducting the experiment. The design matrix comprises a full replication of $2^4 = 16$ factorial design, plus seven centre points and eight star points. Therefore, experimental design consist of 31 (16+7+8=31) experimental runs. Observations in table 2 are modified to nearest higher values; viz. for module if it is 3.56 then we are choosing 4, similarly for number of teeth also means for 36.67 we are taking as 37. After changing such values of teeth and module we are getting 2 to 4 mm more center distance than we are giving as input. Moreover above observations can be taken readily if our gear material is changed; viz. elasticity, bearing strength, starting torque multiplication factor, factor of safety etc.

3.5 Development of Mathematical Models

Mathematical model developed is given by

Y = f (SD, RP, RG, PO)

where, Y is the measured response (MO = Module, NP = No. of teeth on pinion, NG = No. of teeth on gear) and SD is shaft distance, mm.

RP is rpm of pinion, rev/min

RG is rpm of gear, rev/min

PO is power output, kW

Input factors				Output factors		
SD	RP	RG	PO	MO	NP	NG
225	1,100	275	6	3	31	21
225	1,100	275	8	4	23	91
225	1,100	325	6	3	35	116
225	1,100	325	8	4	26	87
225	1,300	275	6	3	27	24
225	1,300	275	8	4	20	93
225	1,300	325	6	3	31	21
275	1,300	325	8	4	23	91
275	1,100	275	6	3	37	47
275	1,100	275	8	4	28	111
275	1,100	325	6	3	42	42
275	1,100	325	8	4	32	07
275	1,300	275	6	3	33	52
275	1,300	275	8	4	25	114
275	1,300	325	6	3	37	47
275	1,300	325	8	4	28	111
200	1,200	300	7	4	21	81
300	1,200	300	7	3	41	61
250	1,000	300	7	4	29	97
250	1,400	250	7	4	23	03
250	1,200	350	7	4	22	04
250	1,200	300	7	3	38	30
250	1,200	300	5	3	34	34
250	1,200	300	9	4	26	01
250	1,200	300	7	4	26	01
250	1,200	300	7	4	26	01
250	1,200	300	7	4	26	01
250	1,200	300	7	4	26	01
250	1,200	300	7	4	26	01
250	1,200	300	7	4	26	01
250	1,200	300	7	4	26	01

TABLE 2

The second order response surface model for the three selected parameters is given by the equation

$$Y = b_0 + \sum_{i=1}^{3} b_i x_i + \sum_{i=1}^{3} b_{ii} x_i^2 + \sum_{\substack{i,j=1 \\ j \neq i}}^{3} b_{ij} x_i x_j$$

The second order response surface model [12] could be expressed as:

$$Y = b_0 + b_1 SD + b_2 RP + b_3 RG + b_4 PO + b_{12} SD.RP + b_{13} SD.RG + b_{14} SD.PO + b_{23} RP.RG + b_{24} RP.PO + b_{34} RG.PO + b_{11} SD.SD + b_{22} RP.RP + b_{33} RG.RG + b_{44} PO.PO$$

Where, b_0 – Free term coefficient, b_1, b_2, b_3 and b_4 – Linear coefficients, b_{11}, b_{22}, b_{33} and b_{44} – Quadratic coefficients and b_{12}, b_{13} , b_{14} , b_{23}, b_{24} and b_{34} – Interaction coefficients.

3.6 Checking Adequacy of Models

Adequacy of the models [14] was then tested by analysis of variance (ANOVA) given in Table 3. The value of R^2 (adjusted), squared multiple R and F-ratio is high enough to be confident for better fit.

Half (linear) model					
Factors	$\%R^2$	$\%R^2$adj.	SS	SEE	F-ratio
MO	61.4	55.5	4.36	0.324	10.36
NP	66.7	61.5	680.04	3.617	2.99
NG	73.7	69.6	9330.6	11.32	18.19
Full model					
MO	74.1	51.5	5.26	0.339	3.28
NP	76	55.1	775.82	3.91	3.63
NG	83.9	69.8	10625.2	11.29	5.95

TABLE 3

The estimate of error is also seemed to be less. Linear regression models are fitting best than any other model and also reduce cumbersome mathematical calculation for prediction.

3.7 Development of Final Model

After certain experiments, it has been found that linear regression model suits best than full model so we have chosen model given below:

$$Y = b_0 + b_1\,SD + b_2\,RP + b_3\,RG + b_4\,PO$$

Final mathematical models are as below. These models are developed with the help of SYSTAT12 software package [13]. The input design parameters are in their uncoded form.

$$MO = 1.37 - 0.00365\ SD + 0.000091\ RP + 0.00036\ RG + 0.424\ PO$$

$$NP = 36.6 + 0.142\ SD - 0.0195\ RP + 0.0218 RG - 3.80 PO$$

$$NG = 86 + 0.566\ SD + 0.0022\ RP - 0.0940\ RG - 15.0\ PO$$

4. Results and Discussion

Standard error is an inherent error in the model developed which is less in this given model than any other model previously developed. It is necessary to minimize standard error. Higher F-ratio and almost zero P-value in ANOVA (Table 3) describe better conformity of results.

1) Interaction effect on module due to center distance and rpm of pinion:

As shown in Fig. 1, as shaft distance increased, module increases. While if we keep our center distance constant and increase rpm, module line is almost vertical. It means that rather rpm of pinion, shaft distance is a major deciding factor for module.

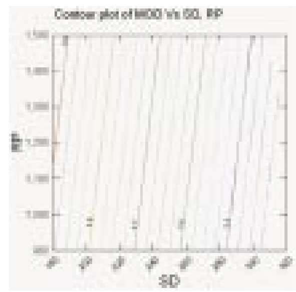

Figure1

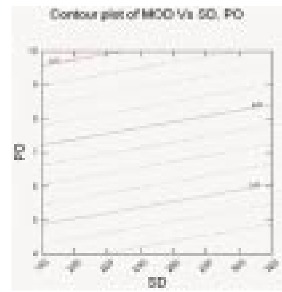

Figure 2

2) Interaction effect on module due to power and shaft distance

Fig 2 shows, module is highly sensitive to power input. Increase in power lead to increase in module (near to 5). If we are keeping power input constant and try to increase our center distance, a little reduction in module is seen. Maximising module needs to increase in power and at the same time decrease shaft distance. Thus both have opposite effect on module.

3) Interaction effect on module due to rpm of pinion and gear

We can see from the Fig. 3, that variation in rpm of pinion and rpm of gear is not affecting so much to the module. Module varies within 3.6 to 3.7 for the entire range of rpm of pinion and gear.

4) Interaction effect on number of teeth due to center distance and power

Fig.4 shows that, if shaft distance is decreased number of teeth decreases. Power is having negative and shaft distance is having positive effect on number of teeth. This means, increase in power will definitely lead to reduce number of teeth while there is a reverse phenomenon when center distance is to be increased.

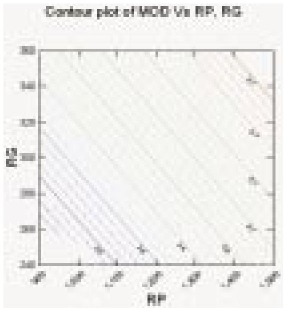

Figure 3

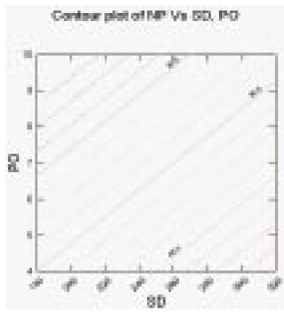

Figure 4

5) *Interaction effect on no. of teeth of pinion due to rpm of pinion and power*

From Fig. 5, we can understand that rpm of pinion and power has negative effect on number of teeth of pinion. Increasing one or both will gradually reduce number of teeth on pinion. It is similar for gear also.

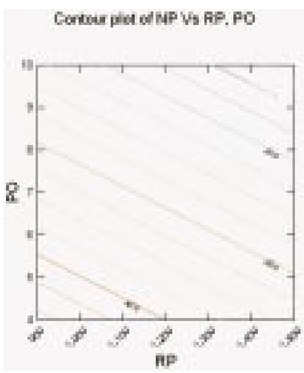

Figure 5

NP actual	By Model.	NG actual	By Model.
28	31.5	113	110.5
24	27.89	107	101.2
26	29	90	86.5

TABLE 6

5. OPTIMIZATION

Single and multi objective optimization were done with the help of MINITAB software package.

Single Objective Optimization

Maximum value of module, minimum number of gear teeth and minimum number of pinion teeth is obtained when single objective optimization problem is considered. Optimum values are given in table 4.

Global or Multi Objective Optimization

Optimum value of maximum module to transmit maximum power and with minimum number of teeth of pinion & gear are given in table 5.

Input				Output		
SD	RP	RG	PO	MO	NP	NG
200	1400	300	9	4.71	–	–
249.85	1400	350	9	–	18.17	–
200	1000	350	9	–	–	8.37

TABLE 4

Input				Output		
SD	RP	RG	PO	MO	NP	NG
200	1000	350	9	4.5	18	77.56

TABLE 5

The range for predicted response shows 95% confident results. If we calculate for shaft distance according to predicted no. of teeth of pinion and gear; [(4.5*20) + (4.5*90)] /2 = 247.5 mm, while given shaft distance is 250 mm. So there is little variation between predicted and actual values. By above results we can conclude that our shaft distance in not be minimum (200 in experiment) every time to optimize gear pair design but at the same time it could transmit maximum power with enough beam strength. Moreover observations are also in modified form means rounding off to nearest higher value as explained previously.

Thus we can say that above results are to be modified to suit standard values of design data books. Table 6 shows comparison of actual values calculated by design and values obtained by model, they have little variation but still under acceptable range.

6. CONCLUSION

1. Mathematical models for module, number of teeth of pinion and gear can give direct values without going for long design process, thus it saves designer's time and efforts.
2. Predicted results are verified by testing, they are reasonably accurate for new observation within range. Optimized parameters for

maximum power transmission for given conditions are SD, RP, RG, PO simultaneously 250 mm, 1400, 350 and 9 kW

3. Designer can easily predict dimensions of spur gear pair. Little modification can finalize design keeping other conditions same.
4. This study and relations can be very useful to designer to have optimum spur gear pair with maximum power output.
5. There are future prospects for optimization of other automotives, machine tools and industrial transmission systems.

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