A Study of Forecasting of Exchange Rates using Non Robust and Robust Estimators

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Abstract

Presence of outliers in exchange rates data is a common feature. In the present study we have tried to construct forecasting models for two exchange rates, that are less sensitive to data contamination by outliers through the Robust estimation techniques namely Least Median Squares (LMS) and Least Trimmed Squares (LTS). The built models are used to assess the predictability of two exchange rates at 1-, 3- and 6- month horizons. The predictive ability of the Robust Linear Autoregressive (RAR) models as compared to that of the Random Walk (RW) and Least Squares (LS) fitted linear autoregressive (AR) models are assessed in terms of forecast accuracies. Further using Diebold-Mariano test the equivalence of forecasts accuracy of two competing models are examined. Using the same criterion the RAR models are also compared. A study on Forecasting models for exchange rates is carried out by Preminger, A and Franck, R, (2007) using RW model and linear AR models fitted by the LS and S- methods of estimation. In the present study we observed that, in general, the performances of robust estimation techniques are better than the LS estimation technique and the overall performance of LTS is better than the LMS and S-estimation techniques.

Keywords: Exchange rates, Forecasting, Outliers, LMS- estimation, LTS-estimation, S-estimation

JEL Classification: C22, C53, C58, F31.

1. Introduction

1.1 Exchange Rates: Importance of Forecasting

International transactions are usually settled in the near future. Exchange rate forecasts are necessary to evaluate the foreign denominated cash flows involved in international transactions. Since the breakdown of the Bretton-Woods system, there has been more interest in predicting exchange rates. The literature shows, however, that exchange rates are largely unpredictable. Thus, exchange rate forecasting is very important to evaluate the benefits and risks attached to the international business environment.

1.2 Review of Literature

In the literature we find many works being carried out related to forecasting of exchange rates. Notable works by Meese and Rogoff (1983a, 1988) showed that the out-of-sample performance of many structural and time series models is no better than that of a simple random walk model. However, the search for a better forecasting model never stops. Researchers venture with more sophisticated modifications of the econometric methodology by allowing different specifications of the dynamics (Edison, 1985) and time varying coefficients (Schinasi & Swamy, 1989; Wolff, 1987), and by incorporating an error correction term (Edison, 1991). Although these improved procedures lead to better performance, a simple random walk process remains the dominant specification for modeling the exchange rate movement. Since these empirical results

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rely mainly on linear time series models, it might be reasonable to conjecture that the unpredictability of exchange rates may be due to the limitations of linear models.

Alternative to this, A variety of parametric nonlinear models such as Autoregressive Conditional Heteroskedasticity (ARCH) (Engle, 1982; Hsieh, 1989a), General Autoregressive Conditional Heteroskedasticity (GARCH) (Bollerslev, 1987), chaotic dynamics (Hsieh, 1991; Peel & Yadav, 1995), and self-exciting threshold autoregression (Chappel, Padmore, Mistry, & Ellis, 1996) models have been proposed and applied to financial forecasting. While these models may be good for a specific data series, they do not have general appeal for other applications. Because there are too many possible nonlinear patterns, the pre specification of the model form restricts the usefulness of these parametric nonlinear models. Further, Diebold and Nason (1990) and Meese and Rogoff (1990) used non-parametric kernel regression and were not able to improve upon a simple random walk model.

In the recent years, another approach to exchange rate forecasting is considered taking into account the presence of outliers in the data as outliers are very common in these data. One such work in this direction is due to Preminger, A and Franck, R, (2007). In the present paper, we have also taken up this approach and tried to focus on the linear time series models which are built using both non robust and robust estimation techniques and studied their relative forecasting performances.

1.2 Presence of Outliers

The presence of the outliers in the exchange rates is very common. Because Exchange rates as well as other financial assets are characterized by dramatic changes over time, as a result of market crashes or rallies, changes in economic policy and business cycles. Such changes can be viewed as introducing outliers (and leverage points) into the time series of interest. The existence of outliers may not be helpful in predicting future returns. Evidence regarding this is well documented in the literature.[1] The presence of outliers can have a dominating and deleterious effect on standard location model estimators such as the least squares regression, the least absolute deviation regression, or even the generalized M-estimators.[2] Furthermore Sakata and White (1995), show that quasi-maximum likelihood (QML) regression estimators are in general vulnerable to outliers.

In the present study data employed are the Japanese Yen (JPY) and the British Pound (GBP) rates against the US dollar. All the data are monthly (from 1971 Jan 01 to 2004 Oct 01) and are obtained from the Federal Reserve Board database. To avoid problems arising from non-stationarity observed in exchange rate data, we compute the differences between natural logarithms of the original exchange rate series. Let e_t denotes the exchange rate at time t, the return series is defined as $y_t = 100 * \log(e_t/e_{t-1})$.

2. Model Building

2.1 The Forecasting Model

In order to obtain forecasts of the return series of the exchange rates we fit regression models based on lagged returns. We consider the linear autoregressive (AR) model, which is given by

$$y_t = \theta_0 + \sum_{j=1}^{p} \theta_j y_{t-1} + \varepsilon_t \qquad (1)$$

2.2 Data Consideration for Estimation

The forecast horizons, h, are chosen to be 1, 3 and, 6 months. In all the experiments, the estimation has been carried out using a "moving regression" approach with fixed window size of the most recent (307-h) observations, based on the estimated model in each window the h-step forecasts were generated. The out-of-sample forecasts include the last 100 observations for all the forecast horizons. The "moving regression" approach is used to handle structural changes in the data, whose presence has been documented.[3] The moving regression approach is less sensitive to possible structural changes in the data and may thus perform better if structural changes did occur.

3. Estimation Techniques

The various estimation techniques used in the study are

- Least Squares (LS)
- Least Median Squares(LMS)

[1] Andersen et al. (2001), Baillie and Bollerslev (1989), Balke, N. S et.al (1994), Dijk, D.V et.al (1999), Sakata, S and White, H (1998)

[2] Maronna et al. (1979) and He (1991)

[3] Gerlach and Petri (1990)

- Least Trimmed Squares(LTS)
- S-Estimation(S)

The Least Squares and S- estimation techniques have been applied to the AR model and the built models have been studied by Preminger, A and Franck, R, (2007). In the present study we use LMS and LTS along with the estimation techniques discussed in Preminger, A and Franck, R, (2007). The AR model is estimated by the Least Squares (LS), which is Non robust to outliers. Due to the sensitivity of the LS-estimation method to outliers, we estimate robust linear AR models (RAR) using the Least Median Squares (LMS), Least Trimmed Squares (LTS) and S-estimation procedures proposed by Rousseeuw, P, (1984, 1983, 1984). The procedures are as described in section 3.1.

3.1 Least Squares (LS) Estimator

It's given by,

$$\min_{\hat{\theta}} \sum_{i=1}^{n} r_i^2 \tag{2}$$

where r^2_i is the square of the i^{th} residual.

3.2 Least Median Squares (LMS)

Replacing the sum in equation (2) by median which is very robust, yields the Least Median Squares (LMS)

$$\min_{\hat{\theta}} \operatorname{median}_i r_i^2 \tag{3}$$

where r^2_i is the square of the ith residual.

3.3 Least Trimmed Squares (LTS)

The LTS estimator is given by,

$$\min_{\hat{\theta}} \sum_{i=1}^{l} \left(r^2\right)_{i:n} \tag{4}$$

where $\left(r^2\right)_{1:n} \leq \left(r^2\right)_{2:n} \leq \ldots \leq \left(r^2\right)_{n:n}$ are the ordered squared residuals.

3.4 S-Estimators

It is defined by,

$$\min_{\hat{\theta}} S(\theta) \tag{5}$$

where S(θ) is a certain type of M estimator of scale on the residuals $r_1(\theta), r_2(\theta), \ldots, r_n(\theta)$.

3.5 Comparison of Robust Estimators

The Robust estimator(s) limits the influence of outlying observations on the estimated parameters. The S-estimators are robust to data contamination by outliers and shown to be asymptotically more efficient than the common least squared (LS) estimators when the data is generated from fat tailed distributions.[4] Rousseeuw and Yohai (1984) have pointed out that these estimators (S-estimators) are much more efficient and computationally less demanding than LTS and LMS estimators. Unlike the LTS and LMS estimators, the S-estimator smoothly down weights outlying observations, which is more natural from a practitioner's point of view.

The LMS estimator can be viewed as a special case of a larger family of estimators, namely the Least Quantile of Squares (LQS) estimators, which are defined by

$$\min \left(r^2\right)_{([(1-\alpha)n]+[\alpha(p+1)]):n} \tag{6}$$

where $0 \leq \alpha \leq 0.05$. For $\alpha = 0.05$, LQS is asymptotically equivalent to the LMS. The breakdown point of the LQS is equal to α as $n \to \infty$.

A disadvantage of the LMS method is its lack of efficiency (because of its $n^{-1/3}$ convergence) when the errors would really be normally distributed. If we consider LMS mainly as a data analytic tool, for which statistical efficiency is not the most important criterion, the fact that the LMS converges like $n^{-1/3}$ does not trouble very much. However, it is not so difficult to improve the efficiency of the LMS estimator. One way to improve the slow rate of convergence of the LMS consists of using a different objective function. Instead of adding all the squared residuals as in LS, one can limit one's attention to a "trimmed" sum of squares. Equation (7) is very similar to LS, the only difference being that the largest squared residuals are not used in the summation, thereby allowing the fit to stay away from the outliers.[5] This quantity is defined as follows: first one orders the squared residuals from smallest to largest, denoted by $(r^2)_{1:n} \leq (r^2)_{2:n} \leq \ldots \leq (r^2)_{n:n}$. Then one can add only the first l of these terms. In this way, Rousseeuw (1983) defined the Least Trimmed Squares

[4] Sakata and White (2001)

[5] Rousseeuw and Leroy (1987)

(LTS) estimator.

$$\min_{\hat{\theta}} \sum_{i=1}^{l} \left(r^2\right)_{i:n} \quad (7)$$

Putting 1 = [n/2] + 1 in (7), the LTS attains the same breakdown point as the LMS. Unlike the slow convergence rate of the LMS, the LTS converges like $n^{-1/2}$, with the same asymptotic efficiency at the normal distribution. The main disadvantage of the LTS is that its objective function requires sorting of the squared residuals, which takes $O(n \log n)$ operations compared with only $O(n)$ operations for the median. The LMS also works in small samples (data set containing 20 points with 6 parameters to be estimated).The LMS, LTS, and S estimators have High Breakdown Point (HBP).[6] In addition to having HBP, the LMS and LTS estimators are regression, scale, and affine equivariant.

- An estimator T is called regression equivariant if $T(\{(x_i, y_i + x_{iv}); i = 1, \ldots, n\}) = T(\{(x_i, y_i); i = 1, \ldots, n\}) + v$, where v is a column vector.
- An estimator T is said to be scale equivariant if $T(\{(xi, c\, y_i); i = 1, \ldots, n\}) = cT(\{(x_i, y_i); i = 1, \ldots, n\})$, where c is any constant.
- One says that T is affine equivariant if $T(\{(x_iA, y_i); i = 1, \ldots, n\}) = A^{-1}T(\{(x_i, y_i); i = 1, \ldots, n\})$,for any nonsingular square matrix A.

Both the LMS and the LTS are defined by minimizing a robust measure of the scatter of the residuals. Generalizing this, Rousseeuw and Yohai (1984) introduced so-called S-estimators. These estimators form another class of high-breakdown affine equivariant estimators with convergence rate $n^{-1/2}$. They are defined by minimization of the dispersion of the residuals:

$$\min s(r_1(\hat{\theta}), r_2(\hat{\theta}), \ldots, r_n(\hat{\theta})) \quad (8)$$

with final scale estimate

$$\hat{\sigma} = s(r_1(\hat{\theta}), r_2(\hat{\theta}), \ldots, r_n(\hat{\theta})) \quad (9)$$

The dispersion $s(r_1(\hat{\theta}), r_2(\hat{\theta}), \ldots, r_n(\hat{\theta}))$ is defined as the solution of

$$\frac{1}{n}\sum_{i=1}^{n} \rho\left(\frac{r_i}{s}\right) = K \quad (10)$$

K is often put equal to $E_{\phi}[\rho]$, where Φ is the standard normal. The function ρ must satisfy the following conditions:

(S_1) ρ is symmetric and continuously differentiable, and $\rho(0) = 0$.

(S_2) There exists $c > 0$ such that ρ is strictly increasing on $[0, c]$ and constant on $[c, \infty)$.

4. Assessing the Forecasting Performances of the Various Estimation Techniques

The out-of-sample predictive performances of the robust and non-robust time series regression model are examined for each exchange rate. Using the measures such as the Root Mean Square Error (RMSE), the Mean Absolute Error (MSE) and the Median of Absolute Deviation (MAD) about the median the forecasting accuracy were assessed. In order to test whether the forecasts from two competing models are equally accurate, the Diebold-Mariano (DM) test is applied. Based on these criteria, all the three above mentioned robust estimation techniques(S, LMS and LTS) were compared. They were also compared with the Non robust (LS and RW) models.

4.1 Measures of Forecasting Accuracy

The RMSE and MAE are the usual measures of prediction error performance, but may be generally less robust to the possible presence of outliers. Hence it is useful to include the MAD statistic, which is not inflated by extreme observations, and gives a more accurate indication of prediction performance for the bulk of the data that is free of outliers. For these three statistics, the lower the output is, the better the forecasting accuracy. These statistics are important measures for forecasting accuracy of the model concerned. Let y_τ and $\hat{y}_\tau$, be the actual return and the predicted return at time τ, respectively, with a forecast period going from $t+1$ to $t+n$. The forecast error statistics are then defined as:

- Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{1}{n}\sum_{\tau=t+1}^{t+n}\left(y_\tau - \hat{y}_\tau\right)^2} \quad (11)$$

[6] [Breakdown Point is defined as the minimum proportion of the data for which contamination by outliers can lead to completely non informative estimation results], i.e., these estimators can resist up to 50% contamination in the data, these estimators are sometimes called as HBP estimators.

- Mean Absolute Error (MSE)

$$MSE = \frac{1}{n}\sum_{\tau=t+1}^{t+n}\left|y_\tau - \hat{y}_\tau\right| \quad (12)$$

- Median of Absolute Deviation (MAD)

$$MAD = \text{median}(\left|y_\tau\right| - \text{median}(y_\tau)) \quad (13)$$

4.2 Testing the Equality of Forecasts from Two Competing Models

Diebold-Mariano (DM) test: In order to test whether the forecasts from two competing models are equally accurate, we use the Diebold and Mariano (1995) (DM) test. This statistic is designed as follows: let us assume that a pair of models produce the h-step ahead forecast errors $\left\{\hat{\varepsilon}^{(1)}_{t+h/t}, \hat{\varepsilon}^{(2)}_{t+h/t}\right\}$ and that the quality of the forecast errors is measured by a specified loss function of the $g\left(\hat{\varepsilon}_{t+h/t}\right)$ forecast error. We can define the "loss differential" between the two competing forecasts as $d_t = g\left(\hat{\varepsilon}^{(1)}_{t+h/t}\right) - g\left(\hat{\varepsilon}^{(2)}_{t+h/t}\right)$. The test is then based on the following large sample statistic:

$$DM = \frac{\bar{d}}{\sqrt{T^{-1}2\pi \hat{f}_{\bar{d}}(0)}} \overset{a}{\sim} N(0,1) \quad (14)$$

where $\bar{d}$ is the sample average of d_t and $2\pi \hat{f}_{\bar{d}}(0)$ is the spectral density of frequency zero which is estimated in the usual way as a two-sided weighted sum of available autocorrelations. We define our loss functions as $d_t = \left(\hat{\varepsilon}^{(1)}_{t+h/t}\right)^2 - \left(\hat{\varepsilon}^{(2)}_{t+h/t}\right)^2$ for the MSE test, and $d_t = \left|\hat{\varepsilon}^{(1)}_{t+h/t}\right| - \left|\hat{\varepsilon}^{(2)}_{t+h/t}\right|$ for MAE test.

5. Results

Using the forecasting model described in section 2.1, considering the data as described in section 2.2 and using a Non robust (LS) and three robust estimating techniques described in section 3, the out-of sample forecasts are obtained for the two exchange rates[7]. The results are as reported in tables I and III for Great Britain Pound. Similarly, tables II and IV present results of Japanese Yen. For the Diebold-Mariano (DM) test we display the p-values, defined as the significance levels at which the null hypothesis under investigation can be rejected. In each table, panels A, B, and C respectively report the results for all the models at the 1-, 3-, and 6-month forecast horizons. Within each panel of Tables I and II, columns (2) to (4) report the RMSE, MAE and MAD forecast accuracy measures. In calculating the DM statistic, the null hypothesis of equal predictive ability is related to the five benchmark models: the RW, AR, RARS, RARLMS, and RARLTS[8] models, i.e., pair wise comparisons are made. The test results are presented in Tables III and IV. We respectively report in columns (2) and (3) of each panel the results of the DM test under the null hypothesis that the square forecast error and the absolute forecast error produced by the RW are smaller than those obtained using each other model. Columns (4) and (5), columns (6) and (7), columns (8) and (9), and columns (10) and (11) are organized in the same manner and show the test results when the benchmark models are respectively the AR, RARS, RARLMS, and RARLTS models.

The forecast accuracy measures, which are presented in Tables I and II for each currency, indicate that the out-of-sample performances of each model vary with the forecast horizon. At the one-month horizon, for GBP based on RMSE and MAE criterion AR model is better whereas with respect to MAD, RARLMS is better. Among the robust models, performance of RARLMS is better. In the case of JPY, the performance of robust models is superior over the RW and AR models. With respect to RMSE, MAD, and MAE, the robust models RARS, RARLMS, and RARLTS are respectively better than other models. Now coming on to the three-month horizon, the AR and RARS models perform better with respect to RMSE and MAD respectively in GBP and JPY. With respect to MAE, the RARLMS and AR models perform better in GBP and JPY respectively. At the six-month horizon, for GBP, the RARLTS out performs all the models considered in the

[7] Programs have been developed to carry out the above work using R-software.

[8] Robust Model built using S- , Least Median Squares (LMS) - , and Least Trimmed Squares (LTS) - estimation techniques are abbreviated as RARS, RARLMS, and RARLTS respectively.

study with respect to all criteria. In the case of JPY, with respect to each criterion, the robust models outperform the RW and AR models and further all the robust models perform equally well.

In the present study the overall performance of the robust models in terms of all the forecast measures are superior over the non robust models. Throughout the study we have observed that RW is out performed in terms of forecast measures by other models. We can also notice that as the horizon increases the performance of robust models gets better as compared to the non robust models. Further if we consider the overall performance among the robust models for both the currencies, the RARLTS performance is superior.

Table 1. Out-of-sample Forecasting Performances of Models for Great Britain Pound (GBP)

MODELS	*RMSE*	*MAD*	*MAE*
Panel A: 1 month horizon			
RW	2.378	1.316	1.869
AR	1.841*	1.231	1.465*
RARS	1.902	1.214	1.511
RARLMS	1.857	1.211*	1.473
RARLTS	1.942	1.261	1.532
Panel A: 3 month horizon			
RW	2.643	1.806	2.089
AR	1.819*	1.236	1.451
RARS	1.834	1.187 *	1.461
RARLMS	1.822	1.234	1.454*
RARLTS	1.855	1.231	1.481
Panel A: 6 month horizon			
RW	2.487	1.747	2.066
AR	1.812	1.234	1.445
RARS	1.809	1.250	1.442
RARLMS	1.811	1.249	1.443
RARLTS	1.808*	1.234*	1.439*

Note: RMSE, MAD and MSE are the forecast accuracy measures as defined in Section 4.1. The RW and AR respectively refer to the Random Walk and Autoregressive models. RARS, RARLMS, and RARLTS refer to the AR model which is robust with estimation techniques S-, Least Median Squares, and Least Trimmed Squares respectively. The asterisks (*) represent the smallest values over each column for the forecast accuracy measures (columns (2) - (4)).

Table 2. Out-of-sample Forecasting Performances of Models for Japan Yen (JPY)

MODELS	*RMSE*	*MAD*	*MAE*
Panel A: 1 month horizon			
RW	3.241	2.230	2.561
AR	2.615	1.740	2.155
RARS	2.598*	1.779	2.117
RARLMS	2.619	1.627*	2.125
RARLTS	2.616	1.787	2.105*
Panel A: 3 month horizon			
RW	3.700	2.467	2.868
AR	2.727*	1.851	2.127*
RARS	2.728	1.819*	2.134
RARLMS	2.735	1.848	2.144
RARLTS	2.731	1.821	2.136
Panel A: 6 month horizon			
RW	4.108	2.301	3.072
AR	2.734	1.822*	2.144
RARS	2.731*	1.822*	2.143*
RARLMS	2.731*	1.822*	2.143*
RARLTS	2.731*	1.822*	2.143*

Note: RMSE, MAD and MSE are the forecast accuracy measures as defined in Section 4.1. The RW and AR respectively refer to the Random Walk and autoregressive models. RARS, RARLMS, and RARLTS refer to the AR model which is robust with estimation techniques S-, Least Median Squares, and Least Trimmed Squares respectively. The asterisks (*) represent the smallest values over each column for the forecast accuracy measures (columns (2) - (4)).

Tables III and IV display the results of Diebold-Mariano (DM) test where the RW, AR, RARS, RAALMS, and RARLTS models are compared to each of the other models considered in the study. In each table, columns (2) and (3) present the test results for the RW, where a p-value no greater than 0.05 indicates that the RW yield a lower forecast error (in terms of squared error or absolute error) relative to the competing model at 5% significance level, while a p-value no smaller than 0.95 means that the benchmark model produces a higher forecast error at 5% level. The same interpretation is given for the p-values reported in columns (4)-(11).

The test results for the GBP at the 1-month horizon indicate that we reject the hypothesis of equal accuracy between the RW and the other models. Further it is noted that the p-values are all greater than 0.95, which indicates that the RW model yields a greater forecast error (in terms of squared error and absolute error) as compared to the other models. The situation is similar even in the case of JPY. Similarly, in the test results for the GBP, we observe that in terms of MSE, the RAR models as a group is significantly better than the AR model (at the 5% significant level) and also the RAR models are better than the AR model in terms of MAE except for RARLMS which is equal to AR model. Where as in the case of JPY, there is no significant difference between the AR and RAR models.

Further for GBP currency when the comparison is made among the robust models, the RARS model yield a lower forecast error (in terms of squared error and absolute error) relative to the RARLMS, where as it yields a higher forecast error relative to the RARLTS in terms of squared error and it is equal to RARLTS in terms of absolute error. For the case of JPY, there is no significant difference between the RARS and other two RAR (RARLMS and RARLTS) models. And between the RARLMS and RARLTS, the RARLTS yield lower forecast error (in terms of squared error and absolute error). It is clear from the DM test results that the performance of RARLTS is superior when compared to other RAR models in the 1-month horizon.

Coming on to the 3-month horizon, the performance of RW model behaves alike as in the case of 1-month horizon with respect to both the currencies. In comparison between the AR and RAR models, we found that except for RARLMS (in terms of MAE AR is superior for JPY) there is no significant difference between AR and any of the RAR models in both the currencies. A comparison among the robust models reveals that there is no significant difference among them with respect to forecast errors for both the currencies except in one case where RARS is better than RARLMS.

Table 3. **Diebold-Mariano Test on the GBP: Comparisons between the Random Walk Model, AR Model, and the Robust AR Models and All Other Models**

MODELS	*RW AS BENCHMARK*		*AR AS BENCHMARK*		*S AS BENCHMARK*		*LMS AS BENCHMARK*		*LTS AS BENCHMARK*	
	RMSE	*MAE*	*RMSE*	*MAE*	*RMSE*	*MAE*	*RMSE*	*MAE*	*RMSE*	*MAE*
Panel A: 1 month horizon										
RW	-	-	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AR	1	1	-	-	0993	0.985	0.955	0.807	0.995	0.975
RARS	1	1	0.007	0.015	-	-	0.994	0.995	0.017	0.113
RARLMS	1	1	0.045	0.193	0.006	0.005	-	-	0.003	0.012
RARLTS	1	1	0.005	0.025	0.983	0.887	0.997	0.988	-	-
Panel B: 3 month horizon										
RW	-	1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AR	1	1	-	-	0.836	0.770	0.749	0.788	0.903	0.905
RARS	1	1	0.164	0.230	-	-	0.169	0.266	0.903	0.905
RARLMS 1	1	1	0.251	0.212	0.831	0.734	-	-	0.920	0.915
RARLTS 1	1	1	0.097	0.095	0.097	0.095	0.08	0.085	-	-
Panel C: 6 month horizon										
RW	-	-	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AR	1	1	-	-	0.061	0.227	0.000	0.052	0.085	0.106
RARS	1	1	0.939	0.773	-	-	0.755	0.626	0.085	0.106
RARLMS 1	1	1	1.000	0.948	0.245	0.374	-	-	0.238	0.200
RARLTS 1	1	1	0.915	0.894	0.915	0.894	0.762	0.800	-	-

Note: The p-values for Diebold-Mariano (1995) test when the benchmark models are the random walk, AR, ARS, ARLMS and ARLTS models, respectively. For each test we consider the MAE and MSE loss functions. P-values no greater the 0.05 indicate that the benchmark model yields a lower forecast error (in terms of squared error or absolute error) relative to the competing model at 5% significance level, while p-values no smaller than 0.95 mean that the benchmark model produces a higher forecast error at the 5% level.

In the case of 6-month horizon, the behavior of the RW model is again same as in 1- and 3- month horizons. For GBP, when the AR model is compared with RAR models, RAR models outperform the AR model (with respect to RMSE). For JPY there is no difference between AR and other RAR models with respect to MAE, further there is no significant difference between AR and other RAR models except for RARLMS (AR is statistically inferior). A comparison among the robust models reveals that there is no significant difference among them with respect to forecast errors for both the currencies except in one case where RARLTS is better than RARS.

Table 4. **Diebold-Mariano Test on the JPY: Comparisons between the Random Walk Model, AR Model, and the Robust AR Models and All Other Models**

MODELS	*RW AS BENCHMARK*		*AR AS BENCHMARK*		*S AS BENCHMARK*		*LMS AS BENCHMARK*		*LTS AS BENCHMARK*	
	RMSE	*MAE*	*RMSE*	*MAE*	*RMSE*	*MAE*	*RMSE*	*MAE*	*RMSE*	*MAE*
Panel A: 1 month horizon										
RW	-	-	0.003	0.007	0.005	0.008	0.019	0.014	0.011	0.012
AR	0.997	0.993	-	-	0.258	0.056	0.535	0.219	0.505	0.133
RARS	0.995	0.992	0.742	0.944	-	-	0.223	0.371	0.298	0.686
RARLMS	0.991	0.986	0.465	0.781	0.777	0.629	-	-	0.003	0.012
RARLTS	0.989	0.988	0.495	0.867	0.702	0.314	0.997	0.988	-	-
Panel B: 3 month horizon										
RW	-	-	0.000		0.000	0.000	0.000	0.000	0.000	0.000
AR	1	1	-	-	0.572	0.856	0.870	0.982	0.663	0.804
RARS	1	1	0.428	0.144	-	-	0.982	0.968	0.663	0.804
RARLMS	1	1	0.130	0.018	0.018	0.032	-	-	0.131	0.106
RARLTS	1	1	0.337	0.196	0.337	0.196	0.869	0.894	-	-
Panel C: 6 month horizon										
RW	-	-	0.001	0.000	0.001	0.000	0.001	0.000	0.001	0.000
AR	0.999	1	-	-	0.016	0.335	0.018	0.287	0.010	0.237
RARS	0.999	1	0.984	0.665	-	-	0.371	0.139	0.010	0.237
RARLMS	0.999	1	0.982	0.713	0.629	0.861	-	-	0.117	0.174
RARLTS	0.999	1	0.990	0.763	0.990	0.763	0.883	0.826	-	-

Note: The p-values for Diebold-Mariano (1995) test when the benchmark models are the random walk, AR, ARS, ARLMS and ARLTS models, respectively. For each test we consider the MAE and MSE loss functions. P-values no greater the 0.05 indicate that the benchmark model yields a lower forecast error (in terms of squared error or absolute error) relative to the competing model at 5% significance level, while p-values no smaller than 0.95 mean that the benchmark model produces a higher forecast error at the 5% level.

6. Summary and Conclusion

Realizing the importance and growing interest in modeling and forecasting foreign exchange rate movements, we have tried to build models and have assessed their forecasting accuracy. But the presence of outliers in the series and the way of handling those using robust estimating methods makes the study more interesting and challenging. The robust estimation techniques are not very popular among practitioners; one of the reasons for this is that these techniques are more computationally demanding than other estimation methods that are not robust to outliers. In Morgenthaler, S, (2007) the discussants Christophe Croux and Peter Filzmoser says that Analyzing real data examples (rather than showing the effect of robust estimation on simulated data) will again contribute to underline the necessity of robust methods. Hence realizing the importance of robust methods, we have carried out the above work.

In the present study the overall performance of robust methods when compared to the non robust methods are good. It is found that the performances of robust estimation techniques are better than the ordinary least squares estimation technique. It is also been observed that the overall performance of robust estimation technique Least Trimmed Squares (LTS) is better than the Least Median Squares(LMS) and S-estimation techniques. Hence we recommend the practitioners to use robust estimation techniques when they suspect the presence of outliers in the data because Robust Statistical procedures are well-suited to play a leading role in two areas: routine data analyses performed with the help of statistical packages and data mining. Another advantage in using robust estimation techniques is that their performance is almost equal to the other non robust estimating techniques even in the absence of outliers in the data, i.e., for "clean" data the robust method gives approximately the same answer as the classical method, the advantages of robust methods should become immediately clear.

From the economics perspective, the findings in the present study have many implications. Forecasts of exchange rates play a vital role in situations such as taking important decisions and making policies for the betterment of the economic conditions of a firm/ state/country. It has been observed that the volatility in currency markets makes exchange rate forecasting a difficult yet challenging task for multinational firms and portfolio managers. For example in Hedging, Multinational corporations (MNCs) constantly face the decision of whether to hedge future payables or receivables in foreign currencies. Another such a decision taking situation in an MNC is Capital budgeting. Here, When an MNC's parent assesses whether to invest funds in a foreign project, the firm takes into account that the project may periodically require the exchange of currencies. Considering the above the practical value lies in that good forecasts can provide useful information to investors for asset allocation, firms in risk hedging and central banks in policy making. Therefore a model which gives minimum forecast error is always preferred in the above context (domain of economics) of decision making problems over models which give higher forecast error. Generally exchange rate data contains outliers, in such contexts robust models reduces the predictive error primarily by down weighting the outliers and hence can address the business cycle better than non robust models and hence a better planning and decision can be made for the future. Further accurate forecasting of future exchange rates can help firms and portfolio managers to better manage the risk of international transactions and reduce the adverse impact of future currency movement on profitability.

There are few extensions for future study. One can even try with non linear models, which may have accounted due to the presence of outliers in the data. Preminger, A and Franck, R, (2007) have discussed about this aspect. One can even try using various robust estimating techniques discussed in the present study in building non linear forecasting models and compare them with other robust and non robust linear models. It may be even interesting in observing the forecasting accuracy of models built using robust method for detection and estimation of outliers.

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